The Direction of Technical Change in Capital-Resource Economies*

Corrado Di Maria†
CentER, Tilburg University

Simone Valente‡
Institute of Economic Research, ETH Zurich

May 8, 2006

Preliminary: do not quote

Abstract

We analyze a multi-sector growth model with directed technical change where man-made capital and exhaustible resources are essential for production. The relative profitability of factor-specific innovations endogenously determines whether technical progress will be capital- or resource-augmenting. We show that convergence to balanced growth implies zero capital-augmenting innovations: in the long run, the economy exhibits purely resource-augmenting technical change. This result provides sound micro-foundations for the broad class of models of exogenous/endogenous growth where resource-augmenting progress is required to sustain consumption in the long run, contradicting the view that these models are conceptually biased in favour of sustainability.

JEL Codes: O31, O33, O41, Q32.

Keywords: Endogenous Growth, Directed Technical Change, Exhaustible Resources, Sustainability.

1 Introduction

The *New Growth Theory*[1] has formalized the determinants of productivity growth in economies where technological progress results from R&D activity. In this framework, horizontal (vertical) innovations improve the quantity (quality) of intermediate goods, and sustained growth obtains through *endogenous technical change* (ETC hereafter).

In the field of resource economics, this generation of models have been exploited to provide new answers to an old question: the problem of sustaining growth in the presence of natural resource scarcity. A vast body of recent literature extends endogenous

*We thank Daron Acemoglu and Sjak Smulders for their precious help.

†E-mail: C.DiMaria@uvt.nl. Phone: +31134662318. Fax: +31134663042. Address: Tilburg University, Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg (The Netherlands).

‡E-mail: svalente@ethz.ch. Phone: +41446324724. Fax: +41446321362. Address: WIF-ETH Zurich, Zurichbergstrasse 18 - ZUE F15, CH-8032 Zurich (Switzerland).

[1] This strand literature was started by the seminal works of Romer (1987) and (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

growth models to include natural resources as an essential input. The central aim of this literature is to determine whether technical progress is effective in ensuring sustained consumption over the long-run. This issue has been addressed in the endogenous technological change framework by Barbier (1999), Sholz and Ziemes (1999), Groth and Schou (2002), Grimaud and Rougé (2003), amongst others. These contributions present models where

(i) the direction of technical change is exogenous, and

(ii) technical progress is, explicitly or implicitly, resource-augmenting.[2]

It should be stressed that assumption (ii) is crucial with respect to the sustainability problem: in the vast majority of growth models with exhaustible resources, ever-increasing consumption *requires* that the resource-augmenting progress strictly exceed the utility discount rate. The same reasoning underlies neoclassical models of optimal growth, where the rate of resource-saving progress is exogenous. Hence, most contributions in this field share the view that innovations increase, directly or indirectly, the productivity of natural resources. However, to our knowledge, the *existence* of purely resource-augmenting technical progress has not been micro-founded so far. Hence, one may object that the above models are conceptually biased in favor of sustainability: since technological progress may in principle be capital- rather than resource-augmenting, specifications (i)-(ii) might reflect a convenient, but strong assumption.

More recently, three important contributions by Daron Acemoglu (1998, 2002, 2003) developed models with *directed technical change* (DTC), where final output is obtained by means of two inputs, e.g. capital and labor, and technical progress may in principle be either labor- or capital-augmenting, or both. The respective rates of technical progress are determined by the relative profitability of developing factor-specific innovations, so that the direction of technical change is determined endogenously. Hence, DTC models can be considered an up-to-date formalization of the Hicksian notion of *induced innovations* - innovations directed at economizing the use of those factors that become expensive due to changes in their relative prices.[3]

This paper investigates whether, and under what circumstances, technical change is endogenously directed towards resource-augmenting innovations. We tackle the issue in a multi-sector DTC framework, where exhaustible resources and accumulable man-made capital are both essential for production. This allows us to represent in more general terms the so-called *Capital-Resource Economy* - the central paradigm in resource economics since the pioneering contributions of Dasgupta and Heal (1974) and Stiglitz (1974). Elaborating on Acemoglu (2003), we assume an R&D sector where capital- and resource-augmenting innovations increase the number of varieties of factor-specific intermediates. Our main result is that purely resource-augmenting technical change takes place along the balanced growth path: although the rate of capital-augmenting progress may be positive in the short run, it falls to zero as the economy approaches balanced growth.

[2] In section 2 we give a precise definition of implicit and explicit rates of resource-augmenting progress.

[3] See Hicks (1932, p. 124). Early formulations of the Hicksian notion of induced innovations include Kennedy (1964) and Drandakis and Phelps (1965).

The scope of this result is twofold. On the one hand, we provide a micro-foundation for Capital-Resource models featuring resource-augmenting progress, in both the Solow-Ramsey and ETC frameworks: in this perspective, our results contradict the view that such models are too optimistic with respect to sustainability. On the other hand, we show that the Hotelling rule - which characterizes an efficient depletion path for an exhaustible stock of resources - fully supports the balanced growth equilibrium: the possibility of developing resource-augmenting innovations allows the price of raw natural resources to grow indefinitely, without conflicting with stationary prices of intermediate goods in the long run.

The plan of the paper is as follows. Section 2 provides a classification of capital-resource economies in terms of technology specifications, and defines implicit and explicit rates of resource-augmenting technical progress. In section 3, we characterize the balanced growth path of the Capital-Resource economy under directed technical change, and derive the main results. Section 4 concludes.

2 Growth theory and resource economics

The much celebrated *Symposium on the Economics of Exhaustible Resources* is often recalled as the first close encounter between growth theory and resource economics. The Capital-Resource model of Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974) - i.e. an extended neoclassical growth model including exhaustible resources as a production factor - has since been considered a central paradigm in resource economics. More recently, several authors exploited new growth theories to analyze capital-resource economies with endogenous technical change: see e.g.Barbier (1999), Sholz and Ziemes (1999), Groth and Schou (2002), Grimaud and Rougé (2003), Bretschger and Smulders (2003).

A central aim of this literature is to determine whether, and under what circumstances, technical progress is effective in ensuring sustained consumption (Bretschger 2005). In this regard, the common denominator of both early and recent models is that a strictly positive rate of *resource-augmenting progress* is necessary to obtain non-declining consumption in the long run. We used italics in order to stress that the type of technological progress is a crucial element in Capital-Resource economies: from the perspective of sustainability, the 'direction' of technical change (whether it is resource-augmenting or capital-augmenting) is even more important than its 'nature' (i.e., whether it is exogenous or endogenous). To clarify this point, consider the following technologies:

$$Y(t) = F(K(t), M(t)R(t)), \tag{1}$$

$$Y(t) = A(t)K(t)^{\alpha_1} R(t)^{\alpha_2}, \tag{2}$$

where Y is output, K is man-made capital, R is an exhaustible resource extracted from a finite stock, F is concave and homogeneous of degree one, and $\alpha_1 + \alpha_2 \leq 1$. Technology (1) features an *explicit rate* of resource-augmenting progress equal to $\dot{M}/M$: the underlying assumption is that the economy develops resource-saving techniques that directly increase the productivity of R. Specification (2) combines the Cobb-Douglas form with disembodied technical progress: the Hicks neutral rate is equal to $\dot{A}/A$.

Firstly, consider the neoclassical framework: in this case, technology (1) exhibits $M(t) = e^{\eta t}$, with $\eta > 0$ exogenous and constant. Then, if cosumption obeys the standard

Keynes-Ramsey rule, a necessary condition for sustained consumption in the long run is $\rho \leq \eta$, where ρ is the utility discount rate.[4] This is a generalization of a well-known result by Stiglitz (1974), who instead assumed technology (2) setting $A(t) = e^{\omega t}$ with $\omega > 0$ exogenous and constant. In this case, the necessary condition for non-declining consumption becomes $\rho \leq \omega/\alpha_2$. Hence, from the perspective of sustainability conditions, what is crucial is not the total effect of technical change on output levels (ω) but rather its resource-saving effect.[5] Indeed, technology (2) can be rewritten as $Y = K^{\alpha_1}\left(e^{(\omega/\alpha_2)t}R\right)^{\alpha_2}$, where (ω/α_2) is the *implicit rate* of resource-augmenting progress. This implies that assuming disembodied progress in association with a Cobb-Douglas form is not innocuous for the problem at hand: under specification (2), technical change is indirectly resource-augmenting.

The same reasoning applies with respect to ETC models, where $\dot{M}/M$ or $\dot{A}/A$ are determined endogenously by R&D activity. On the one hand, sustained consumption still requires that the resource-augmenting rate be at least equal to the discount rate: see e.g. Amigues, Grimaud, and Moreaux (2004). On the other hand, also in this framework, most technology specifications fall in either category (1) or (2). For example, technical progress is explicitly resource-augmenting in Amigues, Grimaud, and Moreaux (2004), whereas Aghion and Howitt (1998, Ch. 5), Barbier (1999), Sholz and Ziemes (1999), and Grimaud and Rougé (2003) assume variants of the Cobb-Douglas form (2).[6]

Hence, the common denominator of capital-resource models is that technological progress is, explicitly or implicitly, resource-augmenting by assumption. But is this assumption plausible? In principle, one might object, technical progress can be purely capital-augmenting instead. For example, suppose that $Y = \Upsilon(NK, R)$, where N represents purely capital-augmenting progress and Υ exhibits an elasticity of substitution different from unity. In this case, the production function does not allow for implicit resource-augmenting progress, and prospects for sustainability change dramatically. It follows from these considerations that a crucial issue is to determine whether (1)-(2) exhibit sound microeconomic foundations: if not, all mentioned contributions are conceptually biased in favor of sustainability because technologies (1) and (2) reflect a convenient, but strong assumption.

Tackling this issue requires assuming that the direction of technical change is endogenous. In the context of multi-sector economies, the DTC framework has been developed by Acemoglu (1998, 2002, 2003), who assumes that the rates of capital- and labor-augmenting technical change are respectively determined by the relative profitability of factor-specific innovations. In particular, Acemoglu (2003) shows that a typical Capital-Labor economy exhibits purely labor-augmenting progress under directed technical change. In the field of environmental economics, models with DTC are analysed by André and Smulders (2005), Di Maria and Smulders (2004) and Di Maria and van der Werf (2005): Di Maria and Smulders (2004) study the role of endogenous technology in explaining cross-country differences in pollution and the pollution haven

[4] See Valente (2005). The same technology is assumed in Gaitan and Roe (2005).

[5] Actually, Stiglitz (1974) considers $Y = K(t)^{\alpha_1} R(t)^{\alpha_2} L(t)^{\alpha_3} e^{\omega t}$, where L is labor supplied inelastically. Results do not change under specification (2), which is chosen for expositional clarity.

[6] Bretschger and Smulders (2003) assume a peculiar CES technology where innovations are not directly resource-augmenting, but spillovers from capital-augmenting innovations directly affect resource productivity. In this case, resource-augmenting spillovers become necessary to sustain the economy, and the underlying logic is the same.

effect of international trade; Di Maria and van der Werf (2005) analyze carbon leakage effects under directed technical change considering clean versus dirty inputs; André and Smulders (2005) consider a Labor-Resource economy and compare equilibrium dynamics with recent international trends in energy supply and consumption. To our knowledge, however, the existence of purely resource-augmenting technical progress in a Capital-Resource Economy has not been micro-founded so far.

In order to address this point, this paper studies whether, and under what circumstances, R&D activity is endogenously directed towards resource-augmenting innovations, given the alternative of developing capital-augmenting innovations. In particular, we assume a CES technology of the form $Y = F(NK, MR)$ with an elasticity of substitution below unity, and investigate the endogenous dynamics of N and M along the balanced growth path. The main difference with respect to Acemoglu (2003) is that, since we substitute fixed labor with a resource flow extracted from an exhaustible stock, input units and factor rewards (that is, R and resource rents) are necessarily time-varying: the extracting sector exploits the natural stock over an infinite time-horizon, and resource prices therefore obey the Hotelling rule (Hotelling 1931). This implies that we cannot translate *a priori* the result of 'purely labor-augmenting progress' of Acemoglu (2003) into 'purely resource-augmenting progress' in our model, until we prove that the Hotelling rule fully supports the time-paths of intermediate goods prices compatibly with balanced growth. We will show that this is actually the case in our model.

3 The model

The supply-side of the economy consists of five sectors: (i) the final sector assembles capital-intensive and resource-intensive goods ($\tilde{K}$ and $\tilde{R}$). These goods are produced by (ii) competitive firms, using n varieties of capital-specific intermediates ($y_{(j)}^{K}$ with $j \in (0, n]$), and m varieties of resource-specific intermediate goods ($y_{(j)}^{R}$ with $j \in (0, m]$), respectively. Factor-specific intermediates are supplied by (iii) monopolists producing $y_{(j)}^{K}$ by means of available man-made capital (K), and producing $y_{(j)}^{R}$ by means of extracted resource (R); the resource is supplied by (iv) an extracting sector that exploits a finite stock (H) of exhaustible natural capital. Finally, (v) the R&D sector consists of firms that develop capital-augmenting innovations (blueprints that increase n) and firms that develop resource-augmenting innovations (blueprints that increase m). The productivity of R&D firms depends on the amounts of 'scientists' employed in the two subsectors (S^K and S^R, respectively).

Our specifications follow the analysis in Acemoglu (2003): aggregate output Y equals

$$Y = F\left(\tilde{K}, \tilde{R}\right) = \left[\gamma \tilde{K}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\, \tilde{R}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $\gamma \in (0, 1)$ is a weighting parameter, and ε is the (constant) elasticity of substitution between $\tilde{K}$ and $\tilde{R}$. From the point of view of resource economics and sustainability theory, the interesting case is that featuring $\varepsilon < 1$: when resource-intensive goods are essential, natural resource scarcity binds the economy over the entire time-horizon considered, $t \in [0, \infty)$.

Competitive firms produce $\tilde{K}$ and $\tilde{R}$ by means of factor-specific intermediates, $y_{(j)}^{K}$ and $y_{(j)}^{R}$. In each instant t, there are $n(t)$ varieties of $y_{(j)}^{K}$ and $m(t)$ varieties of $y_{(j)}^{R}$, and factor-intensive goods are produced according to technologies

$$\tilde{K} = \left[\int_0^n \left(y_{(j)}^K \right)^\beta dj \right]^{\frac{1}{\beta}} \text{ and } \tilde{R} = \left[\int_0^m \left(y_{(j)}^R \right)^\beta dj \right]^{\frac{1}{\beta}}, \tag{4}$$

where $\beta \in (0,1)$. Intermediates $y_{(j)}^K$ and $y_{(j)}^R$ are supplied by monopolists who hold the relevant patent, and exploit linear technologies

$$y_{(j)}^K = K^{(j)} \text{ and } y_{(j)}^R = R^{(j)}, \tag{5}$$

where $K^{(j)}$ indicates units of man-made capital used to produce $y_{(j)}^K$, and $R^{(j)}$ indicates units of resource used to produce $y_{(j)}^R$.[7] The value of patents held by monopolists equals the present-value stream of instantaneous profits implied by capital- and resource-augmenting innovations (π^K and π^R, respectively), discounted by the interest rate r and the assumed obsolescence (depreciation) rate δ:

$$V^i(t) = \int_t^\infty \pi^i(v)\, e^{-\int_t^v (r(\omega)+\delta)d\omega} dv, \text{ with } i = K, R. \tag{6}$$

For future reference, on the basis of (6) we can define an *index of relative profitability* of the two types of innovations as

$$\Delta(t) \equiv \int_t^\infty \frac{n(v)\,\pi^K(v)}{m(v)\,\pi^R(v)} dv. \tag{7}$$

Denoting aggregate capital by $K(t)$, and the total amount of extracted resource by $R(t)$, market-clearing requires

$$\int_0^{n(t)} K^{(j)}(t)\, dj = K(t) \text{ and } \int_0^{m(t)} R^{(j)}(t)\, dj = R(t). \tag{8}$$

The amount of resource R is supplied by the extracting sector. Denoting the interest rate by r and the resource price by q, the present-discounted value of future profits for the extracting sector is

$$\int_0^\infty q(t)\, R(t)\, e^{-\int_0^t r(v)dv} dt, \tag{9}$$

where we have ruled out extraction costs for simplicity. Assuming that the natural resource is exhaustible, extraction plans face the following constraints:

$$\dot{H}(t) = -R(t) \text{ and } \int_0^\infty R(t)\, dt \leq H(0), \tag{10}$$

where H indicates the resource stock.

[7] It is worth noting, at this point, the role of symmetric technologies for factor-intensive goods and intermediates. In this paper, we are interested in the direction of technical change as driven by the 'general nature' of primary inputs, i.e. reproducibility (of man-made capital) versus exhaustibility (of the natural resource). Symmetric technologies in (4) and (5) are essential to this aim: assuming factor-specific elasticities - setting e.g. $\beta^K \neq \beta^R$ in (4) - or different marginal costs for monopolists in (5) would create trivial distortions in the relative profitability of factor-specific innovations, without addressing the main issue.

In this model, the source of endogenous growth is given by increases in the number of varieties: $\dot{n}(t) > 0$ corresponds to capital-augmenting technical change, and $\dot{m}(t) > 0$ corresponds to resource-augmenting technical change. Increases in varieties are obtained through R&D activity. In this sector, free-entry conditions ensure that firms make zero extra profits. Firms developing capital- and resource-augmenting innovations employ S^K and S^R scientists, respectively. An important assumption is that scientists are fully mobile between the two types of firms: in each instant, scientists can be reallocated between capital- and resource-augmenting activity, according to the relative profitability of the two types of innovations. The technologies for invention are represented by

$$\dot{n}/n = b^K S^K \phi\left(S^K\right) - \delta, \quad (11)$$

$$\dot{m}/m = b^R S^R \phi\left(S^R\right) - \delta, \quad (12)$$

where $\delta > 0$ is the obsolescence rate of both innovations, and b^K and b^R are constant productivity indices. The number of scientists affects the productivity of R&D firms through $S^K \phi\left(S^K\right)$ and $S^R \phi\left(S^R\right)$. The function $\phi(.)$ is assumed to be continuously differentiable and strictly decreasing, such that $\partial\left(S^i \phi\left(S^i\right)\right)/\partial S^i > 0$. On the one hand, assuming $\phi'(.) < 0$ captures *crowding effects* among scientists (when more scientists are employed in one sector, the productivity of each declines); on the other hand, the net effect of a marginal increase in employed scientists on the rate of innovation is positive: $\dot{S}^K > 0$ increases $\dot{n}/n$. Crowding effects are not internalized by R&D firms, so that $b^R \phi\left(S^R\right)$ and $b^K \phi\left(S^K\right)$ are taken as given when firms compete for hiring scientists. We further assume that the number of existing scientists (S) suffices to have a stationary mass of varieties ($\dot{m} = \dot{n} = 0$):

$$S > \bar{S}^K + \bar{S}^R \quad (13)$$

where $\bar{S}^K$ and $\bar{S}^R$ satisfy $b^K \bar{S}^K \phi\left(\bar{S}^K\right) = \delta$ and $b^R \bar{S}^R \phi\left(\bar{S}^R\right) = \delta$ by definition.

To close the model, we consider a representative agent with logarithmic instantaneous preferences, and a constant utility discount rate $\rho > 0$. Assuming unit mass population, and denoting aggregate consumption by C, an optimal consumption path is a plan $\{C(t)\}_{t=0}^{\infty}$ that maximizes

$$\int_0^\infty \log C(t)\, e^{-\rho t} dt, \quad (14)$$

subject to the aggregate wealth constraint

$$\dot{K} = rK + qR + wS - C, \quad (15)$$

where rK is capital income (r is the marginal reward of capital), qR represents resource rents, and w is the wage rate for scientists, so that wS is total labor income. Our results do not change if we substitute logarithmic preferences with a CRRA instantaneous utility function: in (14), the intertemporal elasticity of substitution is set equal to one to simplify the exposition.

3.1 Equilibrium

Denote by p^K and p^R the prices of capital- and resource-intensive goods ($\tilde{K}$, $\tilde{R}$), and the prices of factor-specific intermediates ($y_{(j)}^K$, $y_{(j)}^R$) by $\chi_{(j)}^K$ and $\chi_{(j)}^R$, respectively. An equilibrium of the economy is defined by a vector of price time-paths

$$\left\{ p^K, p^R, \chi^K_{(j)}\Big|_{j=0}^{n}, \chi^R_{(j)}\Big|_{j=0}^{m}, r, q, w \right\}_{t=0}^{\infty}$$

and a sequence of allocations

$$\left\{ \tilde{K}, \tilde{R}, y^K_{(j)}\Big|_{j=0}^{n}, y^R_{(j)}\Big|_{j=0}^{m}, K, R, S^K, S^R, C \right\}_{t=0}^{\infty},$$

such that, for given prices in the respective sectors: consumption and investment plans maximize (14) subject to (15); allocations of capital- and resource-intensive goods maximize final sector profits; allocations of capital- and resource-specific intermediates maximize profits

$$p^K \tilde{K} - \int_0^n \chi^K_{(j)} y^K_{(j)} dj \quad \text{and} \quad p^R \tilde{R} - \int_0^m \chi^R_{(j)} y^R_{(j)} dj \tag{16}$$

subject to (4); allocations of capital and resource inputs maximize monopolistic instantaneous profits

$$\pi^K_{(j)} = \left[\chi^K_{(j)} - r\right] y^K_{(j)} \quad \text{and} \quad \pi^R_{(j)} = \left[\chi^R_{(j)} - q\right] y^R_{(j)} \tag{17}$$

subject to demand schedules for $y^K_{(j)}$ and $y^R_{(j)}$; extracted resource flows maximize (9) subject to (10); scientist allocations S^K and S^R imply zero profits for all R&D firms; and all markets clear.

Setting aggregate output as the numeraire good, the equilibrium is characterized by the following relations. First order conditions for the final sector read

$$p^K = \gamma \left(Y/\tilde{K}\right)^{\frac{1}{\varepsilon}} \quad \text{and} \quad p^R = (1-\gamma)\left(Y/\tilde{R}\right)^{\frac{1}{\varepsilon}}, \tag{18}$$

with price-index normalization implying

$$\left[\gamma^{\varepsilon}\left(p^K\right)^{1-\varepsilon} + (1-\gamma)^{\varepsilon}\left(p^R\right)^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}} = 1. \tag{19}$$

Next, maximization of (16) subject to (4) implies demand schedules for intermediates

$$y^K_{(j)} = \left(\chi^K_{(j)}/p^K\right)^{\frac{1}{\beta-1}} \tilde{K} \quad \text{and} \quad y^R_{(j)} = \left(\chi^R_{(j)}/p^R\right)^{\frac{1}{\beta-1}} \tilde{R}. \tag{20}$$

Monopolists producing factor-specific intermediates maximize (17) taking schedules (20) as given, obtaining first order conditions

$$\chi^K_{(j)} = r\beta^{-1} \quad \text{and} \quad \chi^R_{(j)} = q\beta^{-1}. \tag{21}$$

Expressions (21) imply that equilibrium instantaneous profits $\pi^K_{(j)}$ and $\pi^R_{(j)}$ are invariant across varieties: from the market clearing condition (8), we have

$$y^K_{(j)} = K^{(j)} = K/n \quad \text{and} \quad y^R_{(j)} = R^{(j)} = R/m, \tag{22}$$

so that equilibrium profits read

$$\pi^K = r(1-\beta)(n\beta)^{-1} K \quad \text{and} \quad \pi^R = q(1-\beta)(m\beta)^{-1} R. \tag{23}$$

From (23), we can substitute instantaneous profits and obtain equilibrium present-value streams as

$$V^K(t) = \frac{1-\beta}{\beta}\int_t^{\infty}\frac{K(v)}{n(v)}r(v)\,e^{-\int_t^v(r(\omega)+\delta)d\omega}dv, \tag{24}$$

$$V^R(t) = \frac{1-\beta}{\beta}\int_t^{\infty}\frac{R(v)}{m(v)}q(v)\,e^{-\int_t^v(r(\omega)+\delta)d\omega}dv, \tag{25}$$

As regards resource extraction, maximizing (9) subject to (10) yields the standard Hotelling rule

$$\dot{q}/q = r, \tag{26}$$

which implicitly defines an optimal depletion path where the initial amount of extracted resource is $R(0) = \kappa(H_0, K_0)$ for a given $q(0)$.

In the R&D sector, the value of the marginal innovation in the two types of firms is respectively given by $b^K\phi\left(S^K\right)nV^K$ and $b^R\phi\left(S^R\right)mV^R$. In general, the equilibrium wage rate of scientists is given by

$$w = \max\left\{b^K\phi\left(S^K\right)nV^K, b^R\phi\left(S^R\right)mV^R\right\}, \tag{27}$$

which takes into account possible corner solutions. When equilibrium levels of S^K and S^R are both positive, we have $b^K\phi\left(S^K\right)nV^K = b^R\phi\left(S^R\right)mV^R$ and $S^K + S^K = S$, so that

$$\frac{nV^K}{mV^R} = \frac{b^R\phi\left(S - S^K\right)}{b^K\phi\left(S^K\right)} \tag{28}$$

at any instant in which both types of innovations are developed. Finally, consumption dynamics follow the standard Keynes-Ramsey rule

$$\dot{C}/C = r - \rho. \tag{29}$$

Integrating (4) using (22) we obtain

$$\tilde{K} = n^{\frac{1-\beta}{\beta}}K \quad \text{and} \quad \tilde{R} = m^{\frac{1-\beta}{\beta}}R. \tag{30}$$

Substituting (30) in (20), and using conditions (21) we obtain

$$r = \beta p^K n^{\frac{1-\beta}{\beta}} \quad \text{and} \quad q = \beta p^R m^{\frac{1-\beta}{\beta}}. \tag{31}$$

In order to characterize dynamics, it is useful to define elasticity-adjusted indices of intermediates varieties as $N \equiv n^{\frac{1-\beta}{\beta}}$ and $M \equiv m^{\frac{1-\beta}{\beta}}$. From (30) we can thus rewrite aggregate output $Y = F(\tilde{K}, \tilde{R})$ in equilibrium as

$$Y = F(MK, MR) = \left[\gamma(NK)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(MR)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{32}$$

Expression (32) clarifies the role of innovations in determining the rates of technical progress through expansions of intermediates varieties. For this reason we will refer to $\dot{N}/N$ and $\dot{M}/M$ as the (net) rates of capital-augmenting and resource-augmenting technical progress. Being F homogeneous of degree one, we can define the *augmented capital-resource ratio* as

$$x \equiv NK/MR, \tag{33}$$

and express the augmented output-resource ratio Y/MR in terms of the intensive production function $f(x)$, which exhibits the following properties:

$$Y/MR = f(x) = \left[1-\gamma\left(1-x^{\frac{\varepsilon-1}{\varepsilon}}\right)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (34)$$

$$p^K = f'_x(x) = \gamma\left(f(x)/x\right)^{\frac{1}{\varepsilon}}, \quad (35)$$

$$p^R = f(x) - f'_x(x)\,x = (1-\gamma)\left(f(x)\right)^{\frac{1}{\varepsilon}}. \quad (36)$$

From (35) and (36), we can also define the relative capital share as

$$\sigma \equiv \frac{rK}{qR} = \frac{\gamma}{1-\gamma}x^{\frac{\varepsilon-1}{\varepsilon}} \Rightarrow \partial\sigma/\partial x < 0. \quad (37)$$

Also note that prices p^K and p^R can be expressed as[8]

$$p^K(x) = \left[\gamma^\varepsilon + x^{\frac{1-\varepsilon}{\varepsilon}}(1-\gamma)\gamma^{\varepsilon-1}\right]^{1/(\varepsilon-1)} \Rightarrow \partial p^K/\partial x < 0, \quad (38)$$

$$p^R(x) = \left[x^{\frac{\varepsilon-1}{\varepsilon}}\gamma^\varepsilon(1-\gamma)^{\varepsilon-1} + (1-\gamma)^\varepsilon\right]^{1/(\varepsilon-1)} \Rightarrow \partial p^R/\partial x > 0, \quad (39)$$

where the sign of both derivatives follows from $\varepsilon < 1$. That is, when capital- and resource intensive goods are complements, an increase in the augmented capital-resource ratio (x) corresponds to: a decrease in the relative capital share (σ), a decrease in the price of capital-intensive goods (p^K), and an increase in the price of resource-intensive goods (p^K). On the basis of the above relations, the dynamics of x can be expressed in terms of the two indices of intermediates varieties:

Lemma 1. *In equilibrium, the dynamics of the augmented capital-resource ratio are described by*

$$\dot{x} = \varepsilon\frac{f(x)}{f'_x(x)}\left(f'_x(x)\,\beta N - \frac{\dot{M}}{M}\right). \quad (40)$$

Proof. Differentiate (36) to get

$$\dot{p}^R/p^R = \dot{x}f'_x(x)/\left(\varepsilon f(x)\right). \quad (41)$$

From (31) and (35), the interest rate equals

$$r = f'_x(x)\,\beta N. \quad (42)$$

Differentiating the expression for q in (31) we obtain $\dot{q}/q = (\dot{p}^R/p^R) + (\dot{M}/M)$. Substituting $\dot{p}^R/p^R$ from (41), $\dot{q}/q = r$ from (26), and the interest rate from (42), we obtain the dynamic law (40). □

Equation (40) shows that the augmented capital-resource ratio increases (decreases) when the interest rate exceeds (falls short of) the net rate of resource-augmenting technical change, $\dot{M}/M$. Neoclassical and ETC models with purely resource-augmenting

[8] Expressions (38)-(39) can be derived from price-index normalization. Multiplying both sides of (19) by p^R gives $p^R = \left[\gamma^\varepsilon\left(p^K/p^R\right)^{\varepsilon-1} + (1-\gamma)^\varepsilon\right]^{1/(\varepsilon-1)}$. Substituting from (18) the price ratio $p^K/p^R = \gamma(1-\gamma)^{-1}x^{-(1/\varepsilon)}$ yields (39). Symmetric steps yield (38).

progress can be seen as particular cases of this general rule: the basic difference here is that N and $\dot{M}/M$ are both endogenous. If we normalize $N = 1$ and assume $\dot{M}/M = \eta > 0$ (exogenous constant) in equation (40) we have the dynamic rule for the capital-resource ratio in the Ramsey model with exogenous progress (see Valente (2005, eq.16)). Alternatively, normalising $N = 1$ and keeping $\dot{M}/M$ endogenously determined by R&D activity, we have purely resource-augmenting progress *à la* Amigues, Grimaud, and Moreaux (2004).

3.2 Balanced Growth Path

We begin our characterization of long-run equilibria by considering possible Balanced Growth Paths (BGPs). We will denote by y_∞ the limit $\lim_{t\to\infty} y(t)$, and by y_* the value of y along the balanced growth path, for any variable y.
Following the standard definition, a BGP equilibrium features $(\dot{C}/C)_\infty = g_*$ with g_* finite and constant. We now show that $(\dot{C}/C)_\infty = g_*$ implies a constant augmented capital-resource ratio in the long run. Starting from (40), we have three possible cases regarding the asymptotic value of x: in general, the augmented capital-resource ratio may approach zero ($x_\infty = 0$), diverge to infinity ($x_\infty = \infty$), or converge to a positive steady-state value, $x = \bar{x}$ with $\bar{x} > 0$ a finite constant. The next Proposition establishes that only the third case ($x = \bar{x}$) is compatible with BGP.

Proposition 1. *If $(\dot{C}/C)_\infty = g_*$ finite and constant, then $x_\infty = \bar{x} > 0$ finite and constant.*

Proof. The proof builds on the fact that $x_\infty = 0$ and $x_\infty = \infty$ have the following implications:

$$x_\infty = 0 \Rightarrow S^K_\infty = S \Rightarrow (\dot{n}/n)_\infty = b^K S\phi(S) - \delta \Rightarrow (\dot{m}/m)_\infty = -\delta, \tag{43}$$

$$x_\infty = \infty \Rightarrow S^K_\infty = 0 \Rightarrow (\dot{n}/n)_\infty = -\delta \Rightarrow (\dot{m}/m)_\infty = b^R S\phi(S) - \delta, \tag{44}$$

Expressions (43) and (44) are proved in the Appendix, using the index of relative profitability defined in (7). From (43), if the augmented capital-resource ratio approaches zero, all scientists are employed in developing capital-augmenting innovations, and the number of resource-specific intermediates m will approach zero due to depreciation. From (44), in the opposite case, x diverges to infinity, all scientists are employed in resource-augmenting innovations, and the number of capital-specific intermediates will approach zero in the long run. But neither (43) nor (44) are compatible with BGP. Recalling the Keynes-Ramsey rule (29), having $(\dot{C}/C)_\infty = g_*$ requires a constant interest rate. From (31), $\dot{r}_\infty = 0$ in turn requires

$$\lim_{t\to\infty} \frac{\dot{p}^K(t)}{p^K(t)} = -\lim_{t\to\infty} \frac{\dot{N}(t)}{N(t)}, \tag{45}$$

which implies that $\dot{p}^K_\infty$ and $\dot{N}_\infty$ are either both zero or of opposite sign. First, suppose that $\dot{p}^K_\infty > 0$ and $\dot{N}_\infty < 0$: from (38), $\dot{p}^K_\infty > 0 \Rightarrow \dot{x}_\infty < 0 \Rightarrow x_\infty = 0$; but then, expression (43) would imply $\dot{N}_\infty > 0$, which contradicts the supposition. Second, suppose that $\dot{p}^K_\infty < 0$ and $\dot{N}_\infty > 0$: from (38), $\dot{p}^K_\infty < 0 \Rightarrow \dot{x}_\infty > 0 \Rightarrow x_\infty = \infty$; but then, expression (44) would imply $\dot{N}_\infty < 0$, which contradicts the supposition. Hence, in order to have a constant interest rate we need $\dot{p}^K_\infty = \dot{N}_\infty = 0$, which implies $\dot{x}_\infty = 0$ from (38). Consequently, if the economy converges to BGP, $x_\infty = \bar{x} > 0$ with $\bar{x}$ finite and constant. □

Proposition 1 shows that balanced growth requires $\dot{x}_\infty = 0$ and $\dot{N}_\infty = \dot{n}_\infty = 0$, so that if the economy approaches a BGP equilibrium we have $x_\infty = x_*$ and $N_\infty = N_*$. A constant level of N means that the *net* growth rate of capital-specific intermediates is zero. Note that, due to obsolescence ($\delta > 0$), $\dot{n}_\infty = 0$ does not imply zero R&D activity in capital-augmenting innovations: a positive number of scientists ($S^K_\infty > 0$) must work in the capital-augmenting sector in order to keep n, the number of capital-specific intermediates, constant over time. More important,

Proposition 2. *Convergence to BGP implies purely resource-augmenting technical change, with the net rate $\dot{M}/M$ converging to the equilibrium interest rate:*

$$\lim_{t\to\infty} \frac{\dot{N}(t)}{N(t)} = 0 \quad \textit{and} \quad \lim_{t\to\infty} \frac{\dot{M}(t)}{M(t)} = r_* = f'_x(x^*)\,\beta N^*. \tag{46}$$

Proof. From Proposition 1, balanced growth requires $\dot{p}^K_\infty = \dot{N}_\infty = \dot{x}_\infty = 0$. Substituting (42) in (40) and setting $\dot{x}_\infty = 0$ completes the proof. □

This is the main result of the paper. The intuition for (46) is that balanced growth requires constant prices of both capital- and resource-intensive goods ($\dot{p}^K_\infty = \dot{x}_\infty = 0$ implies, from (36), that $\dot{p}^R_\infty = 0$ as well). On the one hand, since the price of resource-intensive goods is proportional to q/M - from (31) - balanced growth is possible only if the net rate of resource-saving progress exactly offsets the growth in the resource price. On the other hand, efficient exploitation of the exhaustible resource requires the resource price to grow at a rate equal to r by virtue of the Hotelling rule (26), implying $(\dot{M}/M)_\infty = r_*$.[9] Hence, the BGP equilibrium of the economy is characterized by the following dynamics:

$$\begin{aligned}
\dot{\tilde{K}}_*/\tilde{K}_* &= \dot{\tilde{R}}_*/\tilde{R}_* = \dot{Y}_*/Y_* = \dot{C}_*/C_* = r_* - \rho, && (47)\\
\dot{R}_*/R_* &= -\rho, && (48)\\
\dot{m}_*/m_* &= \beta\,(1-\beta)^{-1}\,r_*, && (49)\\
\dot{n}_*/n_* &= 0, && (50)\\
\dot{\pi}^K_*/\pi^K_* &= r_* - \rho, && (51)\\
\dot{\pi}^R_*/\pi^R_* &= \frac{1-2\beta}{1-\beta} r_* - \rho, && (52)
\end{aligned}$$

Substituting (51)-(52) in (24)-(25) we obtain the BGP values of patents: if the economy

[9]Formally, this reasoning provides an equivalent proof of Proposition 2: differentiating q from (31) and substituting the Hotelling rule $\dot{q}/q = r$, we obtain $r = (\dot{p}^R/p^R) + (\dot{M}/M)$. Taking the limit and substituting $\dot{p}^R_\infty = 0$ we obtain (46).

converges to balanced growth, we have

$$V^K(t) = \frac{(1-\beta) r_*}{\beta(\delta+\rho) n_*} \cdot K(t), \tag{53}$$

$$V^R(t) = \frac{1-\beta}{\beta\left(\frac{\beta}{1-\beta} r_* + \delta + \rho\right)} \cdot \frac{q(t) R(t)}{m(t)}, \tag{54}$$

for any sufficiently large t. Equations (53)-(54) imply that both nV^K and mV^R will grow at the balanced rate $r_* - \rho$. Finally, equilibrium in the 'labor market for scientists' requires

$$b^K \phi\left(S_*^K\right) n_* V^K(t) = b^R \phi\left(S_*^R\right) m(t) V^R(t), \tag{55}$$

where $S_*^K = \bar{S}^K$ and $S_*^R = S - \bar{S}^K$.

Since $f_x'(.)$ is homogeneous of degree zero and $\partial\left(S^K \phi\left(S^K\right)\right)/\partial S^K > 0$, a unique couple of values $(x_*, S_*^K = \bar{S}^K)$ satisfies $r_* = f_x'(x^*) \beta N^*$ with N^* determined by S_*^K, and the BGP equilibrium is therefore unique. As regards other possible long-run equilibria, the BGP described above is the only possible long-run equilibrium provided that the economy exhibits non-cyclical paths: in this case, $(\dot{C}/C)_\infty = \infty$ cannot be an equilibrium. The proof is identical to that in Acemoglu (2003), and is reported in the Appendix.

As regards the dynamic stability of the BGP equilibrium, we are able to reduce the set of equilibrium conditions to a linearized three-by-three system of differential equations which includes the dynamics of x, N, and S^K. As shown in the Appendix, in the neighborhood of the steady-state equilibrium $\left(x_*, N_*, S_*^K\right)$ we have

$$\begin{pmatrix} \dot{x}/x \\ \dot{N}/N \\ \dot{S}^K/S^K \end{pmatrix} \simeq \begin{pmatrix} a_{xx} & a_{xN} & a_{xS} \\ 0 & 0 & a_{NS} \\ a_{Sx} & 0 & a_{SS} \end{pmatrix} \times \begin{pmatrix} x \\ N \\ S^K \end{pmatrix}, \tag{56}$$

where $a_{xx} < 0$, $a_{xN} > 0$, $a_{xS} > 0$ in the first line; $a_{NS} > 0$ in the second line; and $a_{SX} > 0$, $a_{SS} > 0$ in the third line. Recalling that $R(0) = \kappa(H_0, K_0)$ implies $x(0) = K_0 (\kappa(H_0, K_0))^{-1}$, local stability requires one positive and two negative roots. Being the determinant of the Jacobian matrix $a_{xN} a_{NS} a_{SX} > 0$, we have either three positive roots, or one positive and two negative (or complex with negative real part) roots. The three roots (λ_i) are also zeros of

$$P(\lambda) = -\lambda^3 + \lambda^2 (a_{xx} + a_{SS}) + \lambda (a_{Sx} a_{xS} - a_{xx} a_{SS}) + a_{Sx} a_{NS} a_{xN} = 0,$$

where $(a_{Sx} a_{xS} - a_{xx} a_{SS}) > 0$ and $a_{Sx} a_{NS} a_{xN} > 0$. Hence, regardless of the sign of $(a_{xx} + a_{SS})$, the polynomial always shows one variation of signs (either -,+,+,+ or -,-,+,+). This implies the existence of one and only one positive root, and thus establishes local stability.

3.3 Remarks

We have formalized directed technical change in a Capital-Resource economy by extending the benchmark DTC model of Acemoglu (2003) to include natural capital. Acemoglu (2003) assumes that final output is a combination of capital-intensive and *labor*-intensive goods, and shows that, when both goods are essential, there exists a unique balanced growth path with purely labor-augmenting technical change. In this paper, raw labor inputs are replaced by resource flows extracted from an exhaustible natural stock. We have shown that the equilibrium time-path of resource prices, which obeys the standard Hotelling rule, fully supports the time-path of intermediate goods prices that is compatible with the BGP equilibrium. In particular, the asymmetric role of the two types of innovation follows immediately from equilibrium conditions (31). Balanced growth typically requires a constant interest rate (the rental price of capital): given that q (the price of natural resource) must grow forever, fulfilling (31) for given prices p^K and p^R requires differentiated innovation rates $\dot{m}/m \neq \dot{n}/n$. As a consequence, in our Capital-Resource economy we were able to find a BGP equilibrium, which is locally stable, and features purely resource-augmenting technical change.

From Proposition 2, the asymptotic rate of resource-augmenting progress exactly equals the interest rate. A similar result can be obtained in the neoclassical framework, but following an inverse logic: for a given exogenous rate of resource-augmenting technical progress η, the marginal product of capital converges to η, determining constant factor shares in the long run (Stiglitz 1974). In the present context, instead, the rate of technical change is endogenous and its behavior complies with the Hicksian principle of induced innovations: technical change *tends to be directed* towards those factors that become expensive, in order to compensate relative scarcity with increased real productivity. As a consequence, balanced growth requires that $\dot{M}/M$ converges to the growth rate of resource price, which is in turn equal to the interest rate.

Two final remarks are as follows. Firstly, the uniqueness and the local stability of the BGP equilibrium hinge on the assumption of poor substitution possibilities: setting $\varepsilon > 1$ leaves room for multiple long-run equilibria, and in particular, the possibility that the economy shifts towards alternative paths along which the net rate of capital-augmenting technical progress is positive (for details, see Acemoglu 2003). However, in the present context, our assumption $\varepsilon < 1$ relies on a precise economic reasoning: natural resource scarcity matters for sustainability to the extent that exhaustible resources are essential for production. Secondly, the necessary condition for non-declining consumption in the long run can be expressed as

$$\left(\frac{1-\beta}{\beta}\right) b^R \left(S_*^R\right) \phi \left(S_*^R\right) \geq \rho + \delta, \tag{57}$$

which is obtained by imposing $(\dot{C}/C)_\infty = (\dot{M}/M)_\infty - \rho \geq 0$ in the BGP equilibrium. From (57), lower monopoly profits for intermediate firms, as well as higher depreciation rates for innovations, reduce prospects for sustained consumption in the long run.

4 Conclusion

The vast majority of capital-resource models assumes that technological progress is, explicitly or implicitly, resource-augmenting. This assumption is necessary to obtain sustained consumption in the long run, but it has not been micro-founded so far. At least in principle, R&D activity can also be directed towards capital-augmenting innovations, leaving room for the possibility that technical change does not exhibit resource-saving properties: in this case, most capital-resource models would be too optimistic with respect to the problem of sustainability, and specifying resource-augmenting progress would be a convenient, but strong assumption. Elaborating on Acemoglu (2003), we addressed the problem in the context of a multi-sector economy with directed technical change, where the respective rates of capital- and resource-augmenting progress are determined endogenously by the relative profitability of factor-specific innovations. We characterized the balanced growth path, showing that the rate of capital-augmenting technical progress tends to zero in the long run, and the economy exhibits purely resource-augmenting progress. This result provides sound microfoundations for the broad class of capital-resource models in both the Solow-Ramsey and the ETC framework, and contradicts the view that such models are conceptually biased in favor sustainability.

We have shown that the net rate of resource-saving progress must equal the interest rate along the balanced growth path. While this confirms a standard result of the neoclassical model, the presence of directed technical change provides a different, and very intuitive explanation for this result. On the one hand, since the natural resource stock is exhaustible, the growth rate of the resource price is exactly equal to the interest rate (Hotelling, 1931). On the other hand, balanced growth requires that the rate of resource-saving progress exactly offset the growth in the resource price: this is in compliance with the view that factor-specific innovations are induced by the need of enhancing the real productivity of scarce resources, in order to compensate for their increased expensiveness (Hicks, 1932). Actually, we do not know whether Hicks and Hotelling had been close friends. But making them meet seventy-five years later was a great pleasure for us.

References

Acemoglu, D. (1998): "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, 113, 1055–1089.

——— (2002): "Directed technical change," *Review of Economic Studies*, 69, 781–809.

——— (2003): "Labour- and Capital-Augmenting Technical Change," *Journal of the European Economic Association*, 1(1), 1–37.

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60, 323–351.

——— (1998): *Endogenous Growth Theory.* MIT Press, Cambridge, MA.

Amigues, J. P., A. Grimaud, and M. Moreaux (2004): "Optimal endogenous sustainability with an exhaustible resource through dedicated R&D," Working paper 04.17.154, LEERNA.

André, F., and S. Smulders (2005): "Energy Use, Endogenous Technical Change and Economic Growth," mimeo, CentER- Tilburg University.

Barbier, E. B. (1999): "Endogenous Growth and Natural Resource Scarcity," *Environmental and Resource Economics*, (14), 51–74.

Bretschger, L. (2005): "Economics of technological change and the natural environment: How effective are innovations as a remedy for resource scarcity?," *Ecological Economics*, 54, 148–163.

Bretschger, L., and S. Smulders (2003): "Sustainability and substitution of exhaustible natural resources. How prices affect long-term R&D-investments," Economics Working Paper Series 03/26, ETH Zurich.

Dasgupta, P., and G. Heal (1974): "The Optimal Depletion of Exhaustible Resources," *Review of Economic Studies*, 41, Symposium on the Economics of Exhaustible Resources, 3–28.

Di Maria, C., and S. Smulders (2004): "Trade Pessimists vs Technology Optimists: Induced Technical Change and Pollution Havens," *Advances in Economic Analysis and Policy*, http://www.bepress.com/bejeap/advances/vol4/iss2/art7.

Di Maria, C., and E. van der Werf (2005): "Carbon Leakage Revisited: Unilateral Climate Policy under Directed Technical Change," Discussion paper, CentER, 68.2005.

Drandakis, E., and E. Phelps (1965): "A Model of Induced Invention, Growth and Distribution," *Economic Journal*, CXXVI, 823–840.

Gaitan, B., and T. L. Roe (2005): "Natural Resource Abundance and Economic Growth in a Two-Country World," Economic Development Center Bulletin 05-1, University of Minnesota.

Grimaud, A., and L. Rougé (2003): "Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policy," *Journal of Environmental Economics and Management*, 45(433-453).

Grossman, G. M., and E. Helpman (1991): *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, MA.

Groth, C., and P. Schou (2002): "Can Non-renewable Resources Alleviate the Knife-edge Character of Endogenous Growth?," *Oxford Economic Papers*, 54, 386–411.

Hicks, J. (1932): *The Theory of Wages*. Macmillan, London.

Hotelling, H. (1931): "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39(2), 137–175.

Kennedy, C. (1964): "Induced Bias in Innovation and the Theory of Distribution," *Economic Journal*, LXXIV, 541–547.

Romer, P. M. (1987): "Growth Based on Increasing Returns Due to Specialization," *American Economic Review*, 77, 56–62.

——— (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98, S71–S102.

Sholz, C., and G. Ziemes (1999): "Exhaustible Resources, Monopolistic Competition, and Endogenous Growth," *Environmental and Resource Economics*, 13, 169–185.

SOLOW, R. (1974): “Intergenerational Equity and Exhaustible Resources,” *Review of Economic Studies*, 41, Symposium on the Economics of Exhaustible Resources, 29–46.

STIGLITZ, J. (1974): “Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths,” *Review of Economic Studies*, 41, Symposium on the Economics of Exhaustible Resources, 123–137.

VALENTE, S. (2005): “Sustainable Development, Renewable Resources and Technological Progress,” *Environmental and Resource Economics*, 30(1), 115–125.

A Appendix

A.1 Proof of expressions (43) and (44)

Results (43) and (44) hold true in a Capital-Labor economy as well, so that the proof is identical to that of Lemma 1 in the Appendix of Acemoglu (2003, p.28-29). We make use of the index of relative profitability $\Delta(t)$ defined in (7), and follow a simple logic: when $x_\infty = 0$, the relative profitability of capital-augmenting innovations grows unboundedly ($\Delta_\infty = \infty$) shifting all scientists into that R&D subsector; symmetrically, $x_\infty = \infty$ implies $\Delta_\infty = 0$, and all scientists will be employed in developing resource-augmenting innovations.

Using (24),(25),(7), (37) and equilibrium conditions of instantaneous profits we have

$$\Delta(t) = \frac{\gamma}{1-\gamma}\int_t^\infty x(v)^{\frac{\varepsilon-1}{\varepsilon}}\,dv. \tag{A.1}$$

Being $\varepsilon < 1$, if $x_\infty = 0$ then $\Delta_\infty = \infty$. From (7) and (27), this will imply $S_\infty^K = S$ and $S_\infty^R = 0$, from which $(\dot{n}/n)_\infty = b^K S\phi(S) - \delta$ and $(\dot{m}/m)_\infty = -\delta$ as in expression (43). Conversely, if $x_\infty = \infty$ then $\Delta_\infty = 0$. From (27) it follows $S_\infty^K = 0$ and $S_\infty^R = S$, and hence $(\dot{n}/n)_\infty = -\delta$ and $(\dot{m}/m)_\infty = b^R S\phi(S) - \delta$ in expression (44).

A.2 Ruling out explosive paths

On the basis of (46), we can exclude the possibility of non-balanced growth paths. Unbounded consumption growth can be ruled out as follows: suppose that $(\dot{C}/C)_\infty = \infty$, which in turn requires $(\dot{Y}/Y)_\infty = \infty$. Then, rewrite (34) as

$$Y(t) = M(t)\,R(t)\left[\gamma x(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.2}$$

Expression (A.2) has the following implications. If $x_\infty = \infty$ then $(x^{\frac{\varepsilon-1}{\varepsilon}})_\infty = 0$, which implies $(\dot{Y}/Y)_\infty = (\dot{M}/M)_\infty + (\dot{R}/R)_\infty < \infty$. Also if $x_\infty = \bar{x}$, where $\bar{x}$ is a finite constant, then $(\dot{Y}/Y)_\infty = (\dot{M}/M)_\infty + (\dot{R}/R)_\infty < \infty$. Finally, if $x_\infty = 0$ we have $(\dot{Y}/Y)_\infty < (\dot{M}/M)_\infty + (\dot{R}/R)_\infty < \infty$. Consequently, $(\dot{Y}/Y)_\infty = \infty$ cannot be an equilibrium, implying that $(\dot{C}/C)_\infty = \infty$ cannot be an equilibrium as well.

A.3 Local stability of the BGP equilibrium

The linearized system (56) is obtained as follows. As regards the first equation, substitute (12) for $\dot{M}/M = (1-\beta)\,\beta^{-1}(\dot{m}/m)$ in (40) to obtain

$$\frac{\dot{x}}{x} = \varepsilon \frac{f(x)}{f_x'(x)\,x}\left[f_x'(x)\,\beta N - \frac{1-\beta}{\beta}\left(b^R\left(S-S^K\right)\phi\left(S-S^K\right)-\delta\right)\right]. \tag{A.3}$$

Differentiating the right hand side of (A.3) with respect to x we have

$$\varepsilon\left[f_x'(x)\,\beta N - \left(1-\frac{f_{xx}''(x)}{f_x'(x)}\right)\left(\dot{M}/M\right)\right]. \tag{A.4}$$

Evaluating (A.4) at the steady-state equilibrium (where $f_x'(x)\,\beta N = \dot{M}/M$ from (40)) we obtain

$$a_{xx} = \varepsilon\frac{1-\beta}{\beta}\left[b^R\left(S-S_*^K\right)\phi\left(S-S_*^K\right)-\delta\right] f(x_*)\, f_{xx}''(x_*), \tag{A.5}$$

where $f_{xx}'' < 0$ implies $a_{xx} < 0$. Differentiating (A.3) with respect to N we have

$$a_{xN} = \varepsilon\beta f(x_*) > 0, \tag{A.6}$$

and with respect to S we have

$$a_{xN} = -\varepsilon\frac{f(x_*)}{f_x'(x_*)}\cdot\left[\partial\left(\dot{M}/M\right)/\partial\left(S^K\right)\right]_{S^K=S_*^K} > 0, \tag{A.7}$$

where the sign comes from $\partial\left(\dot{M}/M\left(S-S^K\right)\right)/\partial S^K < 0$.

The second equation in system (56) follows from (11):

$$\frac{\dot{N}}{N} = \beta(1-\beta)^{-1}\left(b^K S^K\phi\left(S^K\right)-\delta\right), \tag{A.8}$$

which implies $a_{Nx} = a_{NN} = 0$ and, by differentiation with respect to S^K,

$$a_{NS} = \left.\frac{\partial S^K\phi\left(S^K\right)}{\partial S^K}\right|_{S^K=S_*^K} > 0. \tag{A.9}$$

The third equation is obtained as in Acemoglu (2003, p.32). Since $S_*^K > 0$ and $S_*^R > 0$, the equilibrium condition (28) holds in an open set around the BGP equilibrium where both types of innovations are developed. Differentiating (28) and substituting (11)-(12) we have

$$\frac{\dot{S}^K}{S^K} = -\frac{1}{B_1\left(S^K\right)}\left[B_2\left(S^K\right)+B_3\left(S^K\right)\cdot B_4(x)\right], \tag{A.10}$$

where

$$B_1\left(S^K\right) = S^K\left(\frac{\phi'\left(S^K\right)}{\phi\left(S^K\right)}+\frac{\phi'\left(S-S^K\right)}{\phi\left(S-S^K\right)}\right), \tag{A.11}$$

$$B_2\left(S^K\right) = \phi\left(S^K\right)S^K - \phi\left(S-S^K\right)\left(S-S^K\right), \tag{A.12}$$

$$B_3\left(S^K\right) = \frac{(1-\beta)\,\phi\left(S^K\right)}{\beta\phi\left(S-S^K\right)\left[\rho+\delta+\beta\left(r_*-\rho\right)(1-\beta)^{-1}\right]}, \tag{A.13}$$

$$B_4(x) = \sigma(x_*) - \sigma(x), \tag{A.14}$$

where the capital share $\sigma(x)$ is defined in (37) and exhibits $\partial\sigma/\partial x < 0$. Differentiating (A.10) with respect to S^K and x we have

$$\frac{\dot{S}^K}{S^K} \simeq a_{Sx}(x - x_*) + a_{SS}\left(S^K - S_*^K\right) \tag{A.15}$$

where little algebra shows that $a_{Sx} > 0$ and $a_{SS} > 0$.