Appendix: Resources, Innovation and Growth in the Global Economy

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A Agents' Behavior and World General Equilibrium

*Household problem: derivation of (11), (12), (13)***.** In each instant t, each household solves the static problem

$$\max_{\left\{X_i^{Hj},X_i^{Fj}\right\}} \log u^J \text{ s.t. } E^J/L^J = \int_0^{N^H} \left(P_i^H X_i^{Hj}/L^J\right) di + \int_0^{N^F} \left(P_i^F X_i^{Fj}/L^J\right) di,$$

where $J = H, F$, $j = h, f$. Denoting by $\varkappa^H$ the Lagrange multiplier, the first-order conditions in H are

$$X_i^{Hh} = \frac{L^H \left(P_i^H\right)^{-\epsilon} \xi^{\epsilon}}{\left[\varkappa^H \int_0^{N^H} \left(X_i^{Hh}/L^H\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\epsilon}} \text{ and } X_i^{Fh} = \frac{L^H \left(P_i^F\right)^{-\epsilon} (1-\xi)^{\epsilon}}{\left[\varkappa^H \int_0^{N^F} \left(X_i^{Fh}/L^H\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\epsilon}}. \tag{A.1}$$

Multiplying both sides of the first (second) equation by P_i^H (P_i^F), integrating both sides across varieties, and eliminating $\varkappa^H$ by means of the initial expressions in (A.1), we obtain

$$X_i^{Hh} = \frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di} \left(P_i^H\right)^{-\epsilon} \text{ and } X_i^{Fh} = \frac{\int_0^{N^F} P_i^F X_i^{Fh} di}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di} \left(P_i^F\right)^{-\epsilon}. \tag{A.2}$$

Taking the ratio between these two expressions and substituting X_i^{Hh} and X_i^{Fh} by means of (A.1), we have

$$\frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^F} P_i^F X_i^{Fh} di} = \frac{\xi}{1-\xi}. \tag{A.3}$$

Following the same steps for F, we have

$$X_i^{Hf} = \frac{\int_0^{N^H} P_i^H X_i^{Hf} di}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di} \left(P_i^H\right)^{-\epsilon} \text{ and } X_i^{Ff} = \frac{\int_0^{N^F} P_i^F X_i^{Ff} di}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di} \left(P_i^F\right)^{-\epsilon}, \tag{A.4}$$

and

$$\frac{\int_0^{N^H} P_i^H X_i^{Hf} di}{\int_0^{N^F} P_i^F X_i^{Ff} di} = \frac{1-\xi}{\xi}. \tag{A.5}$$

Market clearing yields the values of production in the two countries:

$$Y^H = \int_0^{N^H} P_i^H X_i^{Hh} di + \int_0^{N^H} P_i^H X_i^{Hf} di; \tag{A.6}$$

$$Y^F = \int_0^{N^F} P_i^F X_i^{Fh} di + \int_0^{N^F} P_i^F X_i^{Ff} di. \tag{A.7}$$

From (A.3) and (2), we have $\int_0^{N^H} P_i^H X_i^{Hh} di = \xi E^H$ and $\int_0^{N^F} P_i^F X_i^{Fh} di = (1-\xi) E^H$; from (A.5) and (2), we have $\int_0^{N^H} P_i^H X_i^{Hf} di = (1-\xi) E^F$ and $\int_0^{N^F} P_i^F X_i^{Ff} di = \xi E^F$. Combining these with the constraints (A.6)-(A.7), we obtain (12).

Substituting (A.3) and (A.5) in (1), we obtain the indirect utility

$$\tilde{u}^J \left(E^J/L^J\right) = \log \xi^{\xi} (1-\xi)^{1-\xi} + \log \left(E^J/L^J\right). \tag{A.8}$$

In each country the household chooses the time path of expenditure $\left(E^J/L^J\right)$ that maximizes $\int_0^{\infty} e^{-\rho t} \tilde{u}^J \left(E^J(t)/L^J\right) dt$ subject to the dynamic wealth constraint (4) re-written in terms of assets per capita. The logarithmic form (A.8) implies the standard Keynes-Ramsey rules (11).

From (A.2) and (A.4), we have:

$$\frac{X_i^{Hh}}{X_i^{Hf}} = \frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^H} P_i^H X_i^{Hf} di} = \frac{\xi}{1-\xi} \cdot \frac{E^H}{E^F}; \tag{A.9}$$

$$\frac{X_i^{Fh}}{X_i^{Ff}} = \frac{\int_0^{N^F} P_i^F X_i^{Fh} di}{\int_0^{N^F} P_i^F X_i^{Ff} di} = \frac{1-\xi}{\xi} \cdot \frac{E^H}{E^F}, \tag{A.10}$$

where the last terms follow from the derivation of (12) above. Hence:

$$X_i^{Hh} + X_i^{Hf} = X_i^{Hh}\left[1 + \frac{1-\xi}{\xi}\cdot\frac{E^F}{E^H}\right]; \tag{A.11}$$

$$X_i^{Ff} + X_i^{Fh} = X_i^{Ff}\left[1 + \frac{1-\xi}{\xi}\cdot\frac{E^H}{E^F}\right]. \tag{A.12}$$

Using (A.2) and (A.4) to eliminate, respectively, X_i^{Hh} and X_i^{Ff} from the right-hand sides of (A.11) and (A.12), we obtain:

$$X_i^{Hh} + X_i^{Hf} = \frac{\xi E^H + (1-\xi)E^F}{\int_0^{N^H}\left(P_i^H\right)^{1-\epsilon}di}\left(P_i^H\right)^{-\epsilon} = \left(P_i^H\right)^{-\epsilon}\left[\frac{Y^H}{\int_0^{N^H}\left(P_i^H\right)^{1-\epsilon}di}\right]; \tag{A.13}$$

$$X_i^{Ff} + X_i^{Fh} = \frac{\xi E^F + (1-\xi)E^H}{\int_0^{N^F}\left(P_i^F\right)^{1-\epsilon}di}\left(P_i^F\right)^{-\epsilon} = \left(P_i^F\right)^{-\epsilon}\left[\frac{Y^F}{\int_0^{N^F}\left(P_i^F\right)^{1-\epsilon}di}\right], \tag{A.14}$$

where the left-hand sides represent the total demand for the i-th variety produced in country H and F, respectively. Substituting $X_i^H = X_i^{Hh} + X_i^{Hf}$ and $X_i^F = X_i^{Ff} + X_i^{Fh}$ in (A.13) and (A.14), respectively, we obtain the demand schedule (13). □

Manufacturing firms: the monopolist problem, derivation of (14) and (15). The producer of the i-th variety in country J solves the following problem. Given technology (5), the cost-minimizing conditions over rival inputs, $L_{X_i}^J$ and M_i^J, yield $\frac{W^J}{P_M} = \frac{\sigma}{1-\sigma}\frac{M_i^J}{L_{X_i}^J-\phi}$, which in turn yields total cost

$$W^J L_{X_i}^J + P_M M_i^J = W^J\phi + C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}X_i^J, \tag{A.15}$$

where

$$C_X^J\left(W^J, P_M\right) \equiv \left(P_M\right)^{\sigma}\left(W^J\right)^{1-\sigma}\left[\left(\frac{\sigma}{1-\sigma}\right)^{\sigma} + \left(\frac{1-\sigma}{\sigma}\right)^{1-\sigma}\right] \tag{A.16}$$

is the standard unit-cost function homogeneous of degree one. From (A.15) instantaneous profits $\Pi_{X_i}^J$ read $\left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right]X_i^J - W^J\phi$. Since the monopolist knows the demand schedule (13), instantaneous profits can be written as

$$\Pi_{X_i}^J = \left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right]\eth^j\left(P_i^J\right)^{-\epsilon} - W^J\phi. \tag{A.17}$$

where $\eth^J \equiv [Y^J / \int_0^{N^J} \left(P_i^J\right)^{1-\epsilon} di]$ only contains aggregate variables and is therefore taken as given by the single monopolist. The problem is to maximize $V_i^J(t)$ defined in (6), with instantaneous profits given by (A.17). The problem reduces to a static one, where the first order condition determines the price-setting rule of each monopolist,

$$P_i^J = \frac{\epsilon}{\epsilon - 1} C_X^J \left(W^J, P_M\right) \cdot \left(Z_i^J\right)^{-\theta}, \tag{A.18}$$

which implies a positive mark-up of over the marginal cost. Given (A.16), the conditional factor demands for $L_{X_i}^J$ and M_i^J of each firm are

$$L_{X_i}^J = \phi + \frac{\partial C_X^J \left(W^J, P_M\right)}{\partial W^J} \left(Z_i^J\right)^{-\theta} X_i^J = \phi + (1-\sigma) \frac{C_X^J \left(W^J, P_M\right)}{W^J} \left(Z_i^J\right)^{-\theta} X_i^J, \tag{A.19}$$

$$M_i^J = \frac{\partial C_X^J \left(W^J, P_M\right)}{\partial P_M} \left(Z_i^J\right)^{-\theta} X_i^J = \sigma \frac{C_X^J \left(W^J, P_M\right)}{P_M} \left(Z_i^J\right)^{-\theta} X_i^J. \tag{A.20}$$

Substituting $C_X^J \left(W^J, P_M\right) = \frac{\epsilon-1}{\epsilon} \left(Z_i^J\right)^{\theta} P_i^J$ from (A.18), equations (A.19)-(A.20) imply $W^J L_{X_i}^J = W^J \phi + (1-\sigma) \frac{\epsilon-1}{\epsilon} P_i^J X_i^J$ and $P_M M_i^J = \sigma \frac{\epsilon-1}{\epsilon} P_i^J X_i^J$. Integrating across varieties in both these expressions yields (14).

Denote the rate of return to horizontal innovations as r_N^J. Time-differentiating V_i^J in (6) and imposing symmetry, we have $\frac{\dot{V}_i^J}{V_i^J} = r_N^J + \delta - \frac{\Pi_{X_i}^J}{V_i^J}$ and $\frac{\dot{V}_i^J}{V_i^J} = \frac{\dot{Y}^J}{Y^J} - \frac{\dot{N}^J}{N^J}$. Combining these expressions and solving for r^J gives

$$r_N^J = \frac{\dot{Y}^J}{Y^J} - \frac{\dot{N}^J}{N^J} + \frac{\Pi_{X_i}^J(t)}{V_i^J(t)} - \delta,$$

where we can substitute $\Pi_{X_i}^J(t) = \frac{1}{\epsilon} \frac{Y^J}{N^J} - W^J \phi$ from (A.17)-(A.18), as well as $V_i^J = \beta Y^J / N^J$ from (6), to obtain (15). $\square$

Resource-processing in Home: derivation of (16)-(17). In the resource-processing sector, the cost-minimizing conditions over L_M and R yield $\frac{W^H}{p} = \frac{\varsigma}{1-\varsigma} \left(\frac{R}{L_M}\right)^{\frac{1}{\tau}}$. The associated cost function is

$$C_M \left(W^H, p\right) = \left[(\varsigma)^{\tau} \left(W^H\right)^{1-\tau} + (1-\varsigma)^{\tau} p^{1-\tau}\right]^{\frac{1}{1-\tau}}, \tag{A.21}$$

and the conditional factor demands for raw resource and labor read

$$\begin{aligned} pR &= \frac{\partial C_M\left(W^H,p\right)}{\partial p}\frac{p}{C_M\left(W^H,p\right)}P_M M = S_M^R\left(W^H,p\right)\cdot P_M M, \\ W^H L_M &= \frac{\partial C_M\left(W^H,p\right)}{\partial W^H}\frac{W^H}{C_M\left(W^H,p\right)}P_M M = S_M^L\left(W^H,p\right)\cdot P_M M, \end{aligned}$$

where we have defined the elasticities of $C_M\left(.,.\right)$ to resource price and wage as $S_M^R\left(W^H,p\right)$ and $S_M^L\left(p,W^H\right)$, respectively. Recalling that $S_M^L\left(W^H,p\right) = 1 - S_M^R\left(W^H,p\right)$, the above expressions yield (16). Log-differentiating (A.21) we have (17). □

Proof of Proposition 1. Recalling that the global demand for the intermediate is $M = M^H + M^F$, the first expression in (14) implies $P_M M = \sigma\frac{\epsilon-1}{\epsilon}\left(Y^H+Y^F\right)$. Substituting this equation in the first expression in (16), and imposing the market-clearing condition $R = \Omega$, we obtain

$$p\Omega = S_M^R\left(W^H,p\right)\cdot\sigma\frac{\epsilon-1}{\epsilon}\left(Y^H+Y^F\right). \tag{A.22}$$

Since $(1-\xi)$ is the share of expenditures on imported goods in both countries, the balanced trade condition (10) can be re-written as $P_M M^F = (1-\xi)\,E^H - (1-\xi)\,E^F$. Substituting $P_M M^F = \sigma\frac{\epsilon-1}{\epsilon}Y^F$ from (14), we have

$$\sigma\frac{\epsilon-1}{\epsilon}Y^F = (1-\xi)\,E^H - (1-\xi)\,E^F. \tag{A.23}$$

From (12), substituting $Y^F = \xi E^F + (1-\xi)\,E^H$ in (A.23) we obtain the first expression in (18). Substituting $(E^F/E^H)_*$ back in (A.23) to eliminate E^F we have

$$E^H/Y^F = 1 + \sigma\frac{\epsilon-1}{\epsilon}\frac{\xi}{1-\xi}. \tag{A.24}$$

Combining $(E^F/E^H)_*$ in (18) with (A.24), we obtain $\left(E^F/Y^F\right)_* = 1 - \sigma\frac{\epsilon-1}{\epsilon}$, which is the second expression in (19). From (12), we also have $Y^H = \xi E^H + (1-\xi)\,E^F$, where E^F can be substituted by means of $(E^F/E^H)_*$ in (18) to obtain $\left(E^H/Y^H\right)_*$, that is, the first expression in (19). Combining this result with (A.24) yields the second expression in (18).
□

B Resource Booms in World Equilibrium

Production, Wages and Resource Price: derivation of (20), (21), (22), (23). Equation (20) follows directly from the second expression in (18). Equation (22) is derived as follows. Using $\Pi_M = 0$ and the free-entry condition $N^H V_i^H = W^H \beta Y^H$, we rewrite the wealth constraint (4) as $\frac{\dot{N}^H}{N^H} + \frac{\dot{V}_i^H}{V_i^H} = r^H + \frac{L^H}{\beta Y^H} + \frac{p\Omega}{N^H V_i^H} - \frac{E^H}{N^H V_i^H}$. Using $\frac{\dot{V}_i^H}{V_i^H} = \frac{\dot{Y}^H}{Y^H} - \frac{\dot{N}^H}{N^H}$ from the free entry condition, we obtain

$$\frac{\dot{Y}^H}{Y^H} = r^H + \frac{W^H L^H}{\beta Y^H} + \frac{p\Omega}{\beta Y^H} - \frac{E^H}{\beta Y^H}.$$

Substituting $r^H = (\dot{E}^H/E^H) + \rho$ from (11), and recalling that $\dot{E}^H/E^H = \dot{Y}^H/Y^H$ from (18), we have

$$\frac{E^H}{Y^H} = \beta\rho + \frac{W^H L^H}{Y^H} + \frac{p\Omega}{Y^H}. \tag{B.1}$$

Solving this expression for Y^H yields (22). Equation (21) is obtained following the same steps for country F: since Foreign has no resource endowment, the analogous expression of (B.1) for $J = F$ is $\left(E^F/Y^F\right)_* = \beta\rho + W^F L^F/Y^F$, from which we have (21). Finally, re-writing (A.22) as $p\Omega = S_M^R(1,p) \cdot \sigma\frac{\epsilon-1}{\epsilon}\left(Y^H + Y^F\right)/L^H$, and using (20) to eliminate Y^F, we obtain (23). □

Proof of Proposition 2. Equations (20)-(23) form a static system of four equations in four unknowns determining constant equilibrium values E_*^H, Y_*^H, E_*^F, Y_*^F. Differentiating (24) we obtain $d(\Omega p_*)/d\Omega$ in expression (25). Equation (17) then implies that the price elasticity of demand for the raw resource,

$$\epsilon_M^R(1,p) \equiv 1 - \frac{dS_M^R(1,p_*)}{dp_*}\frac{p_*}{S_M^R(1,p_*)},$$

is less than (greater than, equal to) unity if τ is less than (greater than, equal to) unity. The expressions for $dY_*^H/d\Omega$, $dY_*^F/d\Omega$, $dW_*^F/d\Omega$ in (27) follow directly from (20), (21) and (22). □

Horizontal innovation and resource booms: derivation of (28). Plugging $r_N^J = r = \rho$ and setting $\dot{Y}^J = 0$ in (15), we have

$$\frac{\dot{N}^J(t)}{N^J(t)} = -(\rho + \delta) + \frac{1}{\beta}\left[\frac{1}{\epsilon} - \phi \cdot \frac{N^J(t)\, W^J(t)}{Y^J(t)}\right],$$

which can be re-arranged to yield (28). □

Proof of Proposition 3. The proof follows immediately from Proposition 2: see the text above Proposition 3. □

Reallocation of labor in home: derivation of (31), (32), (33). Denoting total employment in start-up operations by $L_N^J = (\dot{N}^J + \delta N^J) \cdot L_{N_i}^J$, the growth rate of the number of firms implied by the free entry condition (7) is

$$\frac{\dot{N}^J(t)}{N^J(t)} = \frac{W^J(t)\, L_N^J(t)}{\beta Y^J(t)} - \delta. \tag{B.2}$$

where we can substitute $\dot{N}^J(t)/N^J(t)$ by (28) to obtain

$$L_N^J(t) = \frac{1 - \beta\epsilon\rho}{\epsilon} \cdot \frac{Y^J(t)}{W^J(t)} - \phi \cdot N^J(t). \tag{B.3}$$

Setting $J = H$ and $W_*^H = 1$, we obtain (31). Setting $W_*^H = 1$ in the second equation in (14), we obtain (32). Substituting (31)-(32) in the market clearing condition $L_M = L^H - L_X^H - L_N^H$ we obtain (33). □

Total factor productivity, growth and welfare: derivation of (34)-(38). Imposing symmetry in (13) and substituting pricing rule (A.18), we obtain (34). Substituting (34) in (1) yields (35). Using (20) to eliminate from Υ^J the value of manufacturing production in country $K \neq J$, we obtain (36). Defining $\Delta^J \equiv \left(N_*^J / N_0^J\right) - 1$, the innovation rate (29) can be re-written as

$$N^J(t) = N_0^J \frac{1 + \Delta^J}{1 + \Delta^J e^{-\nu \cdot t}} \text{ in each } t. \tag{B.4}$$

By definition of TFP, we have $T^J(t) = \left(Z_i^J(t)\right)^\theta \left(N^J(t)\right)^{\frac{1}{\epsilon-1}}$, where we can substitute (B.4) together with $Z_i^J/Z_i^J = z$, to get

$$\log T^J(t) = \log T^J(0) + \theta\kappa t + \frac{1}{\epsilon-1}\log\left(\frac{1+\Delta^J}{1+\Delta^J e^{-\nu\cdot t}}\right).$$

Without loss of generality, we can approximate $\log\frac{1+\Delta^J}{1+\Delta^J e^{-\nu\cdot t}} \simeq \Delta^J\left(1-e^{-\nu\cdot t}\right)$ and thus write $\log T^J(t)$ as in (37). Substituting (36) and (37) in (35), we obtain (38). □

Proof of Proposition 4. Substituting the equilibrium level of instantaneous utility (38) in the welfare function (3) we have

$$U^J = \int_0^\infty e^{-\rho t}\left[\log\Upsilon_*^J + \xi\log T^J(t) + (1-\xi)\log T^K(t)\right]dt. \tag{B.5}$$

Integration of (37) yields

$$\int_0^\infty \log T^J(t)\, e^{-\rho t}dt = \frac{1}{\rho}\log T^J(0) + \frac{1}{\rho^2}\theta\kappa + \frac{1}{\rho}\cdot\frac{\nu}{(\rho+\nu)(\epsilon-1)}\cdot\Delta^J. \tag{B.6}$$

Substituting (B.6) in (B.5), and setting $\log T^J(0) = 0$ without loss of generality, we obtain (39). □

C The Role of Trade: Introducing Tariffs

Suppose that both countries impose tariffs on imported goods. We denote by α_h (α_f) the ad-valorem tariff imposed by the Home (Foreign) government on the units of imported consumption goods produced by Foreign (Home), and by α_m the ad-valorem tariff imposed by the Foreign government on imported resource-based intermediates produced in Home. The tariffs on manufacturing goods α_h and α_f modify the consumers' expenditure constraints and imply, after utility maximization, the following rules for expenditure allocation

$$Y^H = \xi\cdot E^H + \frac{1-\xi}{1+\alpha_f}\cdot E^F \text{ and } Y^F = \xi\cdot E^F + \frac{1-\xi}{1+\alpha_h}\cdot E^H. \tag{C.1}$$

The tariff on resource-based intermediates α_m, instead, modifies the conditional demand of Foreign so that

$$P_M M^H = \sigma \cdot \frac{\epsilon - 1}{\epsilon} \cdot Y^H \text{ and } P_M M^F (1 + \alpha_m) = \sigma \cdot \frac{\epsilon - 1}{\epsilon} Y^F. \tag{C.2}$$

The balanced trade condition thus reads

$$\underbrace{\frac{\sigma}{1+\alpha_m} \cdot \frac{\epsilon - 1}{\epsilon} Y^F}_{P_M M^F} = \underbrace{\frac{1-\xi}{1+\alpha_h} \cdot E^H}_{\int_0^{N^F} P_i^F X_i^{Fh} di} - \underbrace{\frac{1-\xi}{1+\alpha_f} \cdot E^F}_{\int_0^{N^H} P_i^H X_i^{Hf} di}. \tag{C.3}$$

Following the same steps as in the proof of Proposition 1, we obtain the modified ratios

$$\left(\frac{E^F}{E^H}\right)_* = \frac{1-\xi}{1+\alpha_h} \cdot \frac{1 - \frac{\sigma}{1+\alpha_m} \cdot \frac{\epsilon-1}{\epsilon}}{\frac{\xi}{1+\alpha_m} \sigma \frac{\epsilon-1}{\epsilon} + \frac{1-\xi}{1+\alpha_f}}, \tag{C.4}$$

$$\left(\frac{Y^F}{Y^H}\right)_* = \frac{\xi \frac{\sigma}{1+\alpha_m} \frac{\epsilon-1}{\epsilon} \left[\xi^2 - \frac{(1-\xi)^2}{(1+\alpha_h)\left(1+\alpha_f\right)}\right] + \left(\xi + \frac{1-\xi}{1+\alpha_h}\right) \cdot \frac{1-\xi}{1+\alpha_f}}{\left(\xi + \frac{1-\xi}{1+\alpha_f}\right) \frac{1-\xi}{1+\alpha_h}}, \tag{C.5}$$

$$\left(\frac{E^H}{Y^H}\right)_* = \frac{\frac{\xi}{1+\alpha_m} \sigma \frac{\epsilon-1}{\epsilon} + \frac{1-\xi}{1+\alpha_f}}{\frac{\sigma}{1+\alpha_m} \frac{\epsilon-1}{\epsilon} \left[\xi^2 - \frac{(1-\xi)^2}{(1+\alpha_h)\left(1+\alpha_f\right)}\right] + \frac{1-\xi}{1+\alpha_f} \left(\xi + \frac{1-\xi}{1+\alpha_h}\right)}, \tag{C.6}$$

$$\left(\frac{E^F}{Y^F}\right)_* = \frac{1 - \frac{\sigma}{1+\alpha_m} \cdot \frac{\epsilon-1}{\epsilon}}{\xi + \frac{1-\xi}{1+\alpha_f}}, \tag{C.7}$$

where $\xi^2 - (1-\xi)^2 = 2\left(\xi - \frac{1}{2}\right) > 0$ implies that the term in square brackets is strictly positive,

$$\left[\xi^2 - \frac{(1-\xi)^2}{(1+\alpha_h)(1+\alpha_f)}\right] > 0. \tag{C.8}$$

As a consequence of (C.8), the effects of the tariff on intermediates, α_m, are

$$\frac{\partial\left(E^F/E^H\right)}{\partial \alpha_m} > 0 \quad \frac{\partial\left(Y^F/Y^H\right)}{\partial \alpha_m} < 0 \quad \frac{\partial\left(E^H/Y^H\right)}{\partial \alpha_m} < 0 \quad \frac{\partial\left(E^F/Y^F\right)}{\partial \alpha_m} > 0 \tag{C.9}$$

Rewriting (C.6) as

$$\frac{E^H}{Y^H} = \left[\xi + \frac{(1-\xi)^2}{1+\alpha_h} \cdot \frac{1 - \frac{\sigma}{1+\alpha_m} \cdot \frac{\epsilon-1}{\epsilon}}{\xi \frac{1+\alpha_f}{1+\alpha_m} \sigma \frac{\epsilon-1}{\epsilon} + (1-\xi)}\right]^{-1}, \tag{C.10}$$

we also obtain $\frac{\partial\left(E^H/Y^H\right)}{\partial\alpha_h}>0$ and $\frac{\partial\left(E^H/Y^H\right)}{\partial\alpha_f}>0$. Rewriting (C.5) as

$$\frac{Y^H}{Y^F}=\frac{\left(1+\alpha_h\right)\xi\left(\frac{\xi}{1+\alpha_m}\sigma\frac{\epsilon-1}{\epsilon}+\frac{1-\xi}{1+\alpha_f}\right)+\frac{(1-\xi)^2}{1+\alpha_f}\cdot\left(1-\frac{\sigma}{1+\alpha_m}\cdot\frac{\epsilon-1}{\epsilon}\right)}{(1-\xi)\left(\xi+\frac{1-\xi}{1+\alpha_f}\right)}, \tag{C.11}$$

we have $\frac{\partial\left(Y^F/Y^H\right)}{\partial\alpha_h}>0$. As a consequence, the effects of the tariffs on manufacturing goods, α_h and α_f, read

$$\begin{array}{llll}\frac{\partial\left(E^F/E^H\right)}{\partial\alpha_h}<0 & \frac{\partial\left(Y^F/Y^H\right)}{\partial\alpha_h}>0 & \frac{\partial\left(E^H/Y^H\right)}{\partial\alpha_h}>0 & \frac{\partial\left(E^F/Y^F\right)}{\partial\alpha_h}=0\\ \frac{\partial\left(E^F/E^H\right)}{\partial\alpha_f}>0 & \frac{\partial\left(Y^F/Y^H\right)}{\partial\alpha_f}=? & \frac{\partial\left(E^H/Y^H\right)}{\partial\alpha_f}>0 & \frac{\partial\left(E^F/Y^F\right)}{\partial\alpha_f}>0\end{array} \tag{C.12}$$

where the sign of $\partial\left(Y^F/Y^H\right)/\partial\alpha_f$ is ambiguous.

Notice that, by (C.2), the total value of intermediate production now reads

$$P_MM=P_MM^H+P_MM^F=\sigma\frac{\epsilon-1}{\epsilon}\left(Y^H+\frac{Y^F}{1+\alpha_m}\right), \tag{C.13}$$

On the basis of (C.4)-(C.7) and (C.13), and recalling that $pR=S_M^R\left(W^H,p\right)\cdot P_MM$, we can recalculate the four-equations static system (20)-(23) augmented by the presence of tariffs – and thus determine manufacturing production values, resource price and Foreign wage:

$$\frac{Y^F}{Y^H} = \frac{\xi\frac{\sigma}{1+\alpha_m}\frac{\epsilon-1}{\epsilon}\left[\xi^2-\frac{(1-\xi)^2}{(1+\alpha_h)\left(1+\alpha_f\right)}\right]+\left(\xi+\frac{1-\xi}{1+\alpha_h}\right)\cdot\frac{1-\xi}{1+\alpha_f}}{\left(\xi+\frac{1-\xi}{1+\alpha_f}\right)\frac{1-\xi}{1+\alpha_h}}\equiv\breve{\mu} \tag{C.14}$$

$$W^F = \frac{\left(E^F/Y^F\right)_*-\beta\rho}{L^F}Y^F \tag{C.15}$$

$$Y^H = \frac{L^H+p\Omega}{\left(E^H/Y^H\right)_*-\beta\rho} \tag{C.16}$$

$$p\Omega = S_M^R\left(1,p\right)\sigma\frac{\epsilon-1}{\epsilon}\left(1+\frac{\breve{\mu}}{1+\alpha_m}\right)\cdot Y^H \tag{C.17}$$

where $\breve{\mu}\equiv Y^F/Y^H$ is the production ratio defined in (C.14). In particular, combining (C.17) with (C.16), we obtain

$$p\Omega=\frac{L^H\cdot S_M^R\left(1,p\right)}{\breve{\kappa}-S_M^R\left(1,p\right)},\qquad \breve{\kappa}\equiv\frac{\left(E^H/Y^H\right)_*-\beta\rho}{\sigma\frac{\epsilon-1}{\epsilon}\left(1+\frac{\breve{\mu}}{1+\alpha_m}\right)}, \tag{C.18}$$

which is essentially expression (24) augmented for the presence of tariffs. We can now analyze the comparative static effects of variations in the tariff levels.

Variation in α_h. Suppose that the Home government increases α_h. From (C.9), there is a decrease in $\breve{\mu} \equiv Y^F/Y^H$. From (C.10), there is also an increase in $\left(E^H/Y^H\right)_*$. Hence, the net effect on $\breve{\kappa}$ is positive. This implies a reduction in p in (C.18). Considering (C.16), the reduction in p combined with the increase in $\left(E^H/Y^H\right)_*$ implies a decrease in the value of Home's manufacturing production, Y_*^H. This implies that the asymptotic mass of firms in Home, N_*^H, decreases, that is, a negative effect on TFP growth in Home. Considering Foreign: by (C.9), the increase in α_h does not modify $\left(E^F/Y^F\right)_*$. Hence, considering (C.15), the wage-to-production ratio W^F/Y^F is unaffected and there is no effect on Foreign TFP growth.

Variation in α_f. Suppose that the Foreign government increases α_f. From (C.10), there is an increase in $\left(E^H/Y^H\right)_*$. From (C.11), however, the effect on $\breve{\mu}$ is ambiguous. Hence, the net effect on $\breve{\kappa}$ is ambiguous. This implies ambiguous effects on Home's production and TFP growth. Considering Foreign: by (C.9), the increase in α_f increases $\left(E^F/Y^F\right)_*$. Hence, considering (C.15), the wage-to-production ratio W^F/Y^F increases. This implies a reduction in the asymptotic mass of firms in Foreign, N_*^F, and therefore a negative effect on Foreign TFP growth.

Variation in α_m. Suppose that the Foreign government increases α_m. From (C.9), there is a decrease in $\breve{\mu}$ and a decrease in $\left(E^H/Y^H\right)_*$. Hence, the net effect on $\breve{\kappa}$ is ambiguous. This implies ambiguous effects on Home's production and TFP growth. Now consider (C.15): since $\left(E^F/Y^F\right)_*$ increases (by (C.9)), the wage-to-production ratio in Foreign, W^F/Y^F, increases. This implies a reduction in the asymptotic mass of firms in Foreign, N_*^F, and therefore a negative effect on TFP growth in Foreign.