3. Preferences, Demand, and Welfare

3.1 Utility function

- fundamental assumption:

  consumers buy commodities because these commodities provide satisfaction/ utility

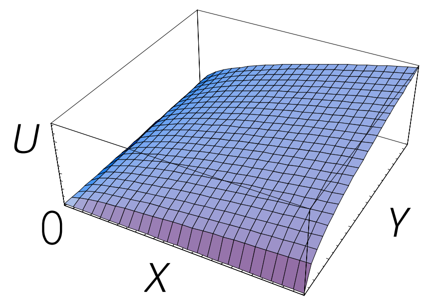
- utility function: relationship between the utility derived from such consumption and the quantities of the commodities consumed: $U = U(X,Y)$

properties of utility function (similar to those of production functions):

- U increases in both arguments

  → a little more of either commodity always increases utility (a consumer is never satiated with a commodity):

$$\frac{dU}{dX} > 0, \quad \frac{dU}{dY} > 0 \quad \text{and} \quad \frac{d^2U}{dX^2} < 0, \quad \frac{d^2U}{dY^2} < 0$$



- difference between utility functions and production functions:

  not possible to assign a specific/numerical value to the utility level associated with a specific consumption bundle (ordinal utility theory)

  But: levels of utility associated with different commodity bundles can be compared: consumer can state whether two bundles yield higher/lower/the same utility

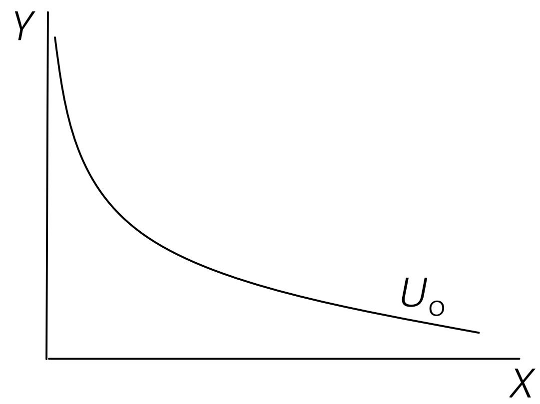
2-dimensional representation of horizontal slice through 'utility hill':

Indifference curves

locus of all combinations of *X* and *Y* that yield the same level of utility,

i.e. locus of all consumption bundles between which consumers are indifferent

(similar to isoquants in production theory, see ch.2, p. 4)

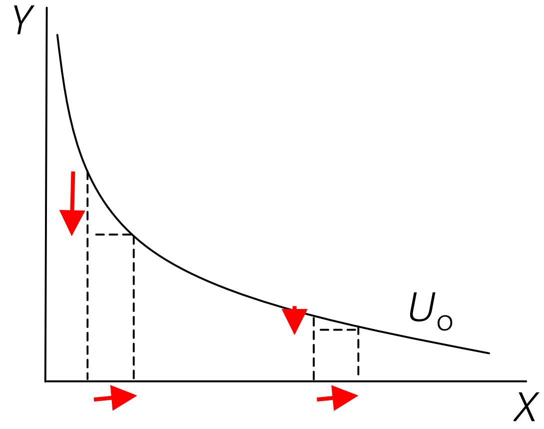


3.2 Characteristics of indifference curves

1. Indifference curves are assumed to be convex to the origin

→ implies: utility associated with a unit of *X* falls relative to utility associated with a unit of *Y* as one moves down the indifference curve (less *Y* and more *X*).

→ increasing *X* constantly by one unit requires smaller and smaller reductions in *Y* to keep utility constant.



- slope of indifference curve: equal to marginal rate of substitution (*MRS*):

$$MRS := -\left.\frac{dY}{dX}\right|_{dU=0}$$

(derivation: same procedure as for *MRS* in production theory, see ch. 2, p. 12)

→ convexity assumption means that the *MRS* is diminishing

2. Utility increases as the individual moves to higher indifference curves (farther away from the origin)

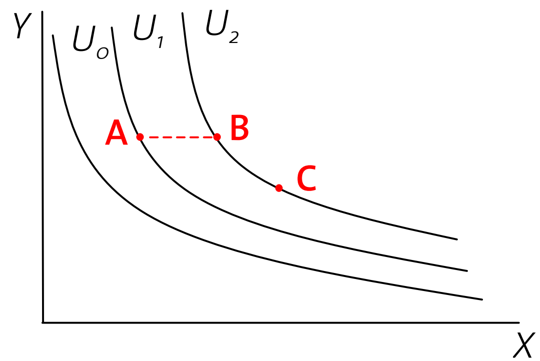
→ results from assumption of non-satiation

using indifference curves, we are able to compare a point which is characterized by more of one good but less of the other good

example: point C is preferred to A

(reasoning: B yields the same utility as C

and B is preferred to A)



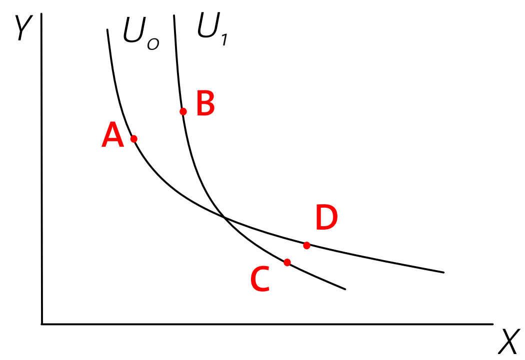
3. No two indifference curves for the same individual can intersect.

→ reason: contradiction

(as at point A a consumer has less of both goods than

at B: $U(A) < U(B) = U(C)$. By the same argument:

$U(C) < U(D) = U(A)$ → contradiction)



3.3 Maximization of utility

Individual consumer's equilibrium:

- consumer's choices, i.e. their demand of X and Y, depend on preferences (as reflected in utility), commodity prices and income

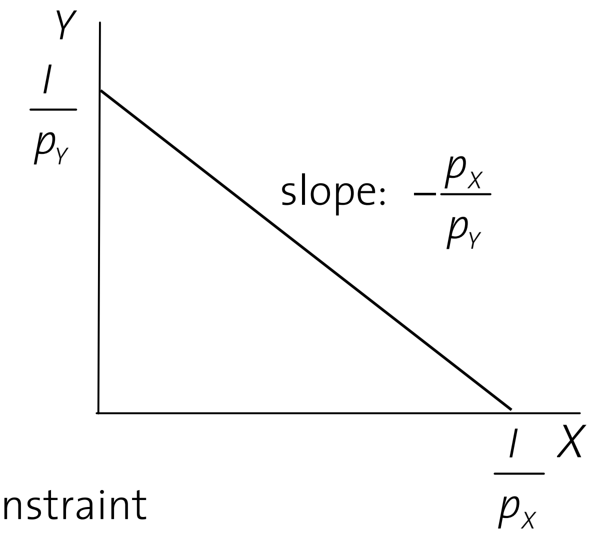
- consumers take prices (p_X and p_Y) and income I as given

- assume, that all income is spent on consumption (i.e. no savings)
  → household's budget constraint:

$$I = p_X X + p_Y Y$$

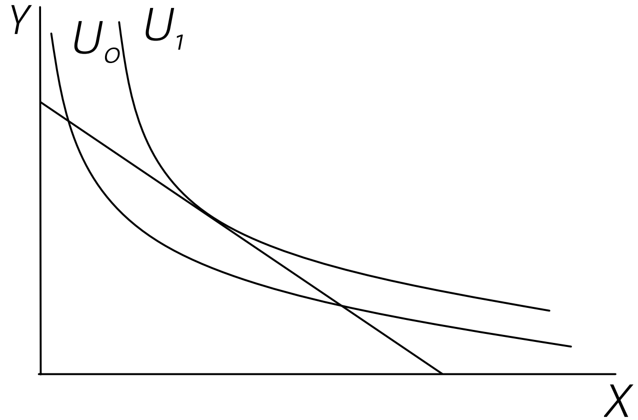
  Graphically:

  - set of maximally feasible consumption points →



- consumers are assumed to maximize utility given their budget constraint

utility maximum: highest level of utility attainable within given budget constraint



→ in utility maximum: tangency between budget constraint and highest possible indifference curve

→ **optimality condition:** slope of budget constraint = slope of indifference curve: $\frac{p_X}{p_Y} = MRS$

(algebraic derivation: maximization of utility subject to budget constraint

→ compare derivation of efficiency condition in production theory, ch. 2, p. 14)

Community equilibrium

Question: what are the conditions that characterize an equilibrium for the entire community?

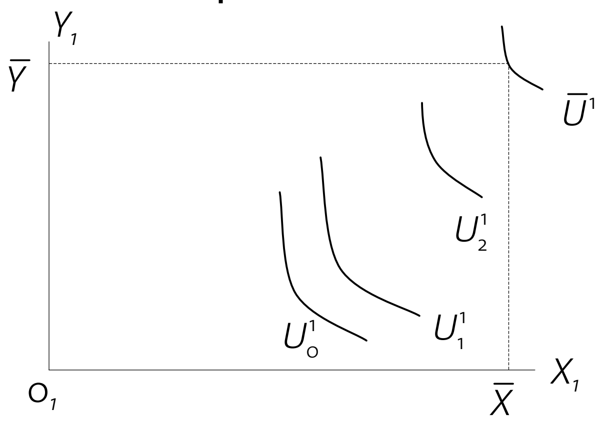
simplest case possible:

- two individuals (labelled 1 and 2) : $U_1 = U_1(X_1, Y_1), \quad U_2 = U_2(X_2, Y_2)$
- amounts of X and Y fixed: $\bar{X} = X_1 + X_2, \quad \bar{Y} = Y_1 + Y_2$

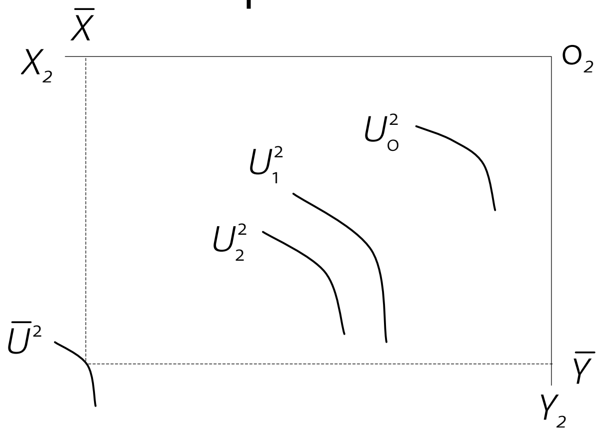
- **optimal allocation** of X and Y between the two individuals:

  allocation for which utility of one consumer cannot be increased without decreasing the utility of the other consumer (Pareto-optimality, Pareto-criterion)

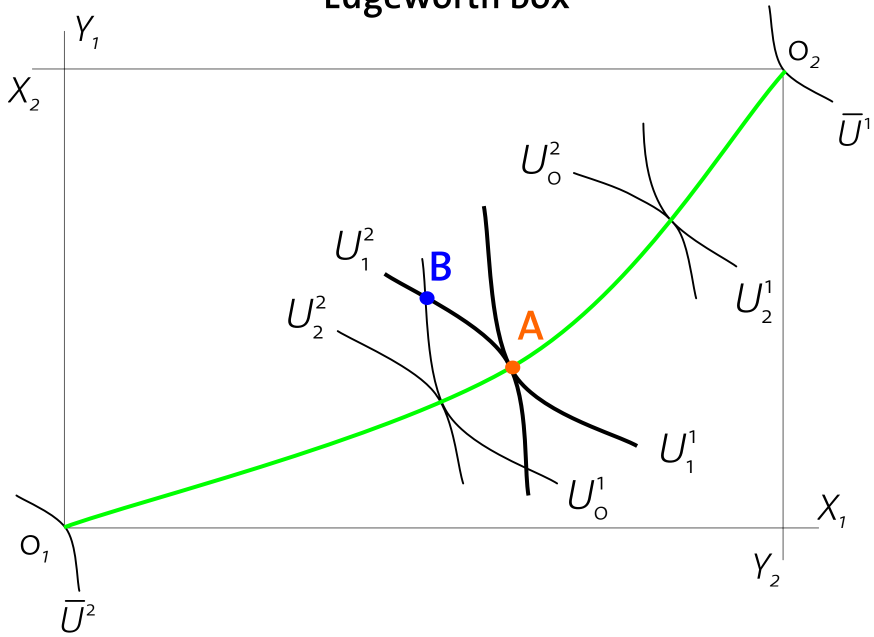
  → derivation (see next page): Edgeworth box (similar to derivation of efficiency locus, see ch. 2, p. 21) consumption individual 1



consumption individual 2



Edgeworth box



contract curve: locus of all Pareto-optimal allocations

(tangency points between the indifference curves for the two individuals; efficient trading locations)

Note: Pareto-criterion is an efficiency condition, has nothing to do with equity (distribution of utility):

→ o_2 is, e.g., Pareto-optimal (on the contract curve) although individual 2 has nothing to consume

- Remember: condition for individual optimum → $\frac{p_X}{p_Y} = MRS$ (for each consumer)
- From Edgeworth box analysis:

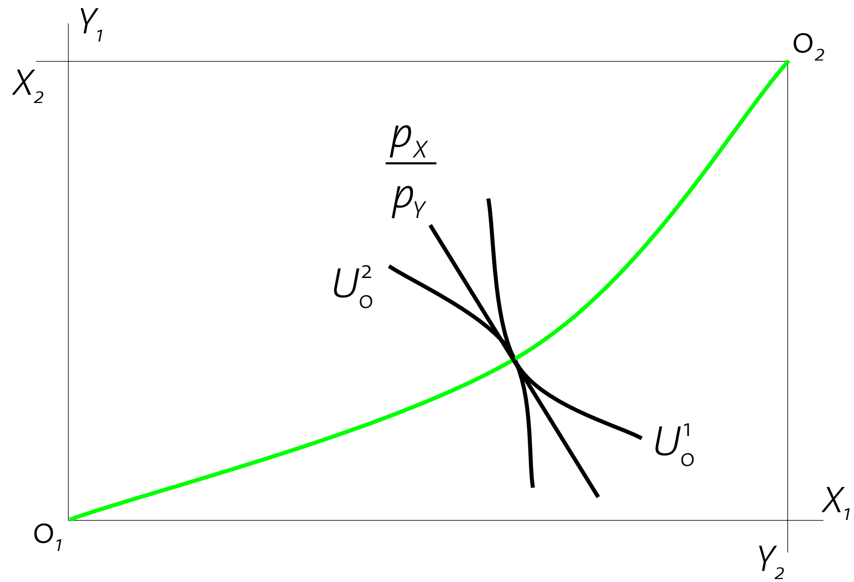
  optimality condition → tangency of indifference curves (slope if indifference curves identical) :

  $\rightarrow MRS_1 = MRS_2$

- consequently condition for Pareto optimality:

  both individuals have to face the same price ratio, such that in the Pareto-optimum:

$$\boxed{\frac{p_X}{p_Y} = MRS_1 = MRS_2}$$



3.4 Aggregating individual preferences

Question: can individual preferences be aggregated to preferences of the entire community?

→ only if certain conditions are met

Consider the following:

- the aggregate demand for a commodity X is given by $X = D(p, I_1, I_2, ..., I_n)$

  with $p = \frac{p_X}{p_Y}$ and I_j is the income of consumer j= 1,...,n (→ n consumers with different income levels)

- existence of aggregate (community) preferences requires
  - demand for every commodity is independent of the distribution of income.

    → in this case, the demand function can be written as: $X = D(p, I)$ with $I = \sum_{j=1}^{n} I_j$

    → aggregate preferences exist whenever a redistribution of income does not affect demand

→ two conditions for which this independency is given:

1. individual preferences are **identical**

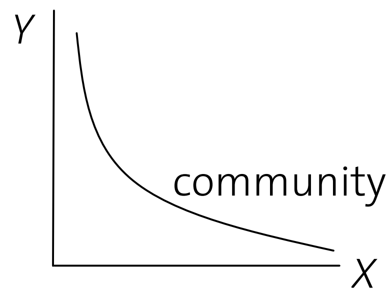
2. individual preferences are **homothetic**

   homothetic functions:

   - more general concept than homogeneity
     (homothetic: all monotonic transformations of homogeneous functions, e.g. $U = a + X^{\alpha_X} Y^{\alpha_Y}$)

   - share important properties of homogeneous functions:
     optimal ratio of consumed goods constant for rising income → expansion path is straight line

→ given that these two conditions hold, individual preferences can be aggregated into community indifference curves



3.5 Interpreting community indifference curves: aggregate demand versus individual welfare

Two interpretations of community indifference curves:

Positive interpretation:

- community indifference curves tell us what the economy will demand under various price and aggregate income combinations

Normative interpretation:

- welfare significance of moving from a lower to a higher curve
- if, e.g., trade policy leads to such a movement, we say that an economy is better off under that policy
- however:
    - the economy might be better off in the sense of reaching a higher (social) indifference curve.

- but: If the new situation is characterized by a different income distribution, it might be that
  - → welfare of individual 1 may rise while welfare of individual 2 may fall
  - → situations not comparable using the Pareto criterion
  - → aggregate welfare effect of trade policy might be positive, but might make some individuals worse off
  - → (notice: if all individuals had identical preferences (utility function): "winners" could compensate "losers" → everybody would be better off)